University of Lakki Marwat

28420 - Lakki Marwat, Khyber Paktunkhwa, Pakistan
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Department of Mathematical Sciences

SCHEME OF STUDIES
M.Phil. / Ph.D. PROGRAMME IN MATHEMATICS

Session 2021 and onwards

HOD, Department of Mathematical Science, ULM **Director Academics, ULM**

SCHEME OF STUDY FOR M.Phil/Ph.D

M. Phil (First Year)

First Semester

Course Code	Course Title	Credit Hours
MATH-XXX	Core Course	3
MATH-XXX	Optional	3
MATH-XXX	Optional	3
MATH-XXX	Optional	3
	Total	12

Second Semester

Course Code	Course Title	Credit Hours
MATH-XXX	Core Course	3
MATH-XXX	Optional	3
MATH-XXX	Optional	3
MATH-XXX	Optional	3
	Total	12

Third & Fourth Semester	**Credit Hours**
MATH- 799 Research Thesis	06

Ph. D (First Year)

First Semester

Course Code	Course Title	Credit Hours
MATH-XXX	Optional	3
MATH-XXX	Optional	3
MATH-XXX	Optional	3
	Total	09

Second Semester

Course Code	Course Title	Credit Hours
MATH-XXX	Optional	3
MATH-XXX	Optional	3

MATH-XXX	Optional	3
	Total	09

Third to Sixth Semester — **Credit Hours**

MATH- 899	Ph. D Thesis	36

Core Courses:

MATH-701	Advanced Algebra	03
MATH-702	Numerical Solution of ODE's	03
MATH-715	Computational fluid dynamics	03
MATH-731	Optimization Theory I	03
MATH-743	Fuzzy Algebra	03
MATH-769	Advanced Mathematical Physics	03
MATH-707	Advanced Ring Theory	03
MATH-739	Viscous fluid I	03
MATH-734	Perturbation Methods	03
MATH-723	Mathematical techniques for Boundary value problems	03
MATH-704	Advanced Group Theory	03
MATH-719	Geometric Function Theory	03
MATH-776	Topics in Numerical Mathematics	03
MATH-786	Semi Ring Theory	03

Optional Courses		**Credit Hours**
MATH-703	Advanced Graph Theory	03
MATH-705	Advanced operation research I	03
MATH-706	Advanced operation research II	03
MATH-708	Advanced Topology	03
MATH-709	Algebraic Topology	03
MATH-710	Advanced Analytical Dynamics I	03
MATH-711	Advanced Analytical Dynamics II	03
MATH-712	Applied Functional Analysis	03
MATH-713	Banach Algebra	03
MATH-714	Bio Mathematics	03
MATH-716	Computational Methods	03

MATH-717	Convolution in geometric function theory	03
MATH-718	Differential subordination theory	03
MATH-720	Integral equations	03
MATH-721	Large scale scientific computation	03
MATH-722	Advance Mathematical methods	03
MATH-724	Multivariate analysis I	03
MATH-725	Multivariate analysis II	03
MATH-726	Numerical Analysis of spectral methods	03
MATH-727	Numerical linear algebra	03
MATH-728	Numerical solutions of integral equations I	03
MATH-729	Numerical solutions of integral equations II	03
MATH-730	Numerical solution of PDEs	03
MATH-732	Optimization Theory II	03
MATH-733	Advanced Partial differential equations	03
MATH-735	Advanced Perturbation Methods	03
MATH-736	Probability and probability distributions I	03
MATH-737	Probability and probability distributions II	03
MATH-738	Semi Group Theory	03
MATH-740	Viscous fluid II	03
MATH-741	Theory of AG-Groupoids	03
MATH-742	Variational Inequalities	03
MATH-744	General Relativity	03
MATH-745	Objectives of Modern Algebra with Applications	03
MATH-746	Advance Functional Analysis	03
MATH-747	Fixed Point Theory	03
MATH-748	Topics in Complex Analysis	03
MATH-749	Mathematical Modelling-I	03
MATH-750	Mathematical Modelling-II	03
MATH-751	Optimal Control	03
MATH-752	Near Rings-I	03
MATH-753	Near Rings-II	03
MATH-754	Several Complex Variable	03
MATH-755	Theory of Group Actions	03
MATH-756	Banach Lattices	03
MATH-757	Linear System Theory	03
MATH-758	Electrodynamics	03
MATH-759	The Classical Theory of Fields	03
MATH-760	Plasma Theory	03
MATH-761	Electrodynamics-I	03
MATH-762	Electrodynamics-II	03
MATH-763	Magneto hydrodynamics-I	03
MATH-764	Magneto hydrodynamics-II	03
MATH-765	Introduction to Algebraic Cryptography	03

MATH-766	La-Semi groups	03
MATH-767	Theory of Group Graphs	03
MATH-768	Lie Algebras	03
MATH-770	Quantum Field Theory	03
MATH-771	Design Theory	03
MATH-772	Computer Graphics	03
MATH-773	Rings and Modules	03
MATH-774	Operator Theory	03
MATH-775	Lattice Theory	03
MATH-777	Topics in Pure Mathematics	03
MATH-778	Topics in Applied Mathematics	03
MATH-779	Nature-Inspired Optimization Techniques	03
MATH-780	Engineering Optimization	03
MATH-781	Multi objective Optimization	03
MATH-782	Game Theory	03
MATH-783	Non-Newtonian Fluid Mechanics	03
MATH-784	Theory of Hyper Structure-I	03
MATH-785	Theory of Hyper Structure-II	03
MATH-786	Advanced Fuzzy Set Theory	03
MATH-787	Introduction to Fractional Calculus	03
MATH-788	Techniques in Fractional Calculus	03
MATH-789	Advanced Mathematical Methods	03
MATH-790	Fixed Point Theory and its Applications	03
MATH-791	Computational Methods in Engineering and Its Boundary value problems	03
MATH-792	Multiple attribute decision making methods And Applications	03
MATH-793	Iterative Approximation Procedures	03

DETAILS OF INDIVIDUAL COURSES FOR M.Phil./Ph.D.

MATH-701 ADVANCED ALGEBRA
Credit Hours: 03+0

Objectives and Outcomes
The goal of the course is to introduce the student with the theory of Advance topics in abstract algebra. The course is designed for the understanding of Field extensions, Galois Theory and Valuation theory which is very important for the new research domain in abstract algebra for example in Algebraic topology. Galois Theory provides a means of proving the impossibility of duplicating the cube, squaring the circle and trisecting a general angle. It also gives beautiful and almost complete algebraic proofs of fundamental Theorems of algebra.

Course Contents
Quadratic forms and modular functions, Interpretation of rings and Ideals, Finite invariants of a field, Hilbert sequence of fields and groups, Artin theorem and symbols, The field extension, Splitting field, Sylow theory, Jordan- Hölder theory, Galois theory, Valuation theory.

RECOMMENDED BOOKS:

1. P. M. Cohn, Algebra Vol. I & II, John Wiley & Sons.
2. Burton, A First Course in Rings & Ideals, Addison Wesley Co.
3. J. Lambek, Lectures on Rings & Modules, Blaisdel.
4. M. S. Atiyab, I. G. Macdonald, Introduction to Commutative Algebra, Addison Wesley Pub. Com., London, 1969.
5. D. E. A. Burton, First Course in Rings and Ideals, Addison Wesley Pub. Company, 1968.
6. O. Mariski, P. Samuel, Commutative Algebra, London, 1963.
7. I. Kaplansky, Commutative Rings University of Chicago Press.
8. F. W. Anderson, K. R. Puller, Rings and Category of Modules, Springer-Verlag.

MATH-703 ADVANCED GRAPH THEORY
Credit Hours: 03+0

Objectives and Outcomes
The aim of the course is to introduce the student to the theory of graphs, particularly algorithmic graph theory. The student will learn a number of standard and powerful algorithms, as well as demonstrating methodologies in graph techniques. In addition the student will be introduced to the use of graphs in the solution of complex problems.

Graph theory has become one of the major tools for the design and analysis of algorithms, as well as the focus of much interest in theoretical computer science.

Course Contents

Review of basic concepts (Vertices, edges, loop, degree, complete graphs, graph isomorphism, adjacency matrices, subgraphs, walks, paths, circuits, The Konigsberg bridge problem, connected graphs, disconnected graphs, and components).
Euler graphs, Euler line, operations on graphs, Hamiltonian paths and circuits, The Travelling salesman problem, Trees, properties of trees, pendant vertices in a tree, distance and centres in a tree, rooted and binary trees, spanning trees, fundamental

circuits, finding all spanning trees of a graph, spanning trees in a weighted graph, Kruskal's algorithm, Prim's algorithm, Boruska's algorithm, Cut-Sets, properties of a cut-set, fundamental cut-sets, connectivity and seperability. Network Flows, network flow problems, 1-Isomorphism, 2-Isomorphism, Planar and Dual graphs, Kuratowski's two graphs, different representations of a planar graph, coloring, partitioning and covering, chromatic partitioning, algorithm for maximum independent set, Matching, Dijkstra algorithm for shortest path, Maximal flow problem, Residual network, Ford-Fulkerson algorithm, Bread-first search algorithm, Depth-first search algorithm.

RECOMMENDED BOOKS

1. R. G. Busacker, T. L. Seaty, Finite graphs and Networks', An introduction with applications', McGraw Hill Book Company.
2. R. J. Wilson, Introduction to Graph Theory, Longman Scientific and technical, 1985.
3. Wai-Kaichen, Applied graph Theory "graphs and Electrical networks, North-Holland Pub. 1976.
4. N. Deo, Graph Theory with Applications to Engineering and Computer Science,Prentice-Hall, INC. Englewood Cliffs, N.J.

MATH-704 ADVANCED GROUP THEORY
Credit Hours: 03+0

Objectives and Outcomes

The aim of the course is to introduce the student with the basic concepts of group theory. The concept of group is fundamental in abstract algebra. Almost all structures in abstract algebra are special cases of groups e.g Galois Theory uses groups to describe the symmetries of the roots of a polynomial, Algebraic Knot Theories takes in a crucial way on classifying spaces of groups. Similarly Algebraic topology is another domain which prominently associates groups to the objects the theory is interested in.

Course Contents

Normal and Subnormal Series, Abelian and Central Series, Direct Products, Finitely Generated Abelian Groups, Splitting Theorems, Solvable and Nilpotent Groups, Commutators Subgroup, Derived Series, The Lower and Upper Central Series, Characterization of Finite Nilpotent Groups, Fitting Subgroup, Frattini Subgroup, Dedekind Groups, Supersolvable Groups, Solvable Groups with Minimal Condition.

Subnormal Subgroups, Minimal Condition on Subnormal Subgroups, The Subnormal Socle, the Wielandt Subgroup and Wielandt Series, T-Groups, Power Automorphisms, Structure and Construction of Finite Soluble T-Groups.

RECOMMENDED BOOKS

1. D.J.S. Robinson, A Course in the Theory of Groups, Graduate Texts in Mathematics 80, Springer, N Y, 1982.
2. K. Doerk, T. Hawkes, Finite Soluble Groups, De Gruyter Expositions in Mathematics 4, Walter De Gruyter, Berlin, 1992.

MATH-705 ADVANCED OPERATION RESEARCH-I
Credit Hours: 03+0

Objectives and Outcomes

This course is intended to provide students with an advance knowledge that can make them appreciate the use of various research operations tools in decision making in organizations. At the end of the Course participants are expected to demonstrate a working knowledge of the various OR /OM tools in making decisions as well as being able to formulate organizational problems into OR models for seeking optimal solutions.

Course Contents

Introduction; Definition and Historical background; or models; Principal Components of decision problems; Type of or models; phases of or study.Linear programming; Graphical solution; Examples of LP applications; The standard form; Basic Solutions and bases; The Simplex method; Special cases in simplex method application; Sensitivity analysis; The Dual problem; Definition of the Dual problem; Primal –Dual relationships; Dual simplex method. Transportation model; Definition and application; solution of the transportation model; The assignment model; The transhipment model.

RECOMMENDED BOOKS

1. H. A. Taha, Operations Research An Introduction, Collier MacMillan, 1982.
2. H. Lieberman, Introduction to operation research, 8th edition, The McGraw Hill Co.

MATH-706 ADVANCED OPERATIONS RESEARCH-II
Credit Hours: 03+0

Objectives and Outcomes

This course will focus on mathematical modelling. A strong emphasis will be given to model formulation. On the methodology side, Linear and Integer Programming techniques will be introduced. At the end of the course, students will have the skills to build their own formulations, to expand existing formulations, to critically evaluate the impact of model assumptions and to choose an appropriate solution technique for a given formulation.

Course Contents

Dynamic programming; Elements of the DP model; Definition of the state and stages; examples of the DP model and computations; problem of dimensionality; solution of linear programming by DP. Game Theory; two-persons Zero-same
Games; mixed strategies; Graphical solution of (2xn) and (mx2) games; Solution of (m×n) Games by Linear programming. Project scheduling by PERT-CPM; Arrow diagram representation; Construction of the time chart; cost consideration in project scheduling; project control. Inventory Model; The general inventory problem and definition; Deterministic (static and dynamic) Inventory models.

RECOMMENDED BOOKS

1. H. A. Taha, Operations Research An Introduction, Collier MacMillan, 1982.
2. H. Lieberman, Introduction to operation research, 8thedition,The McGraw Hill co.

MATH-707 ADVANCED RING THEORY
Credit Hours: 03+0

Objectives and Outcomes

The aim of the course is to familiar the student with the concepts of ring theory. Ring theory is very important in the field of abstract algebra like the concept of vector spaces in linear algebra and functional analysis. Ring theory is the study of rings-algebraic structure in which addition and multiplication are defined and have similar properties to those familiar from the integers. Ring theory studies the structure of rings, their representation, or, in different language, modules, special classes of rings (group rings, division rings) as well as an array of properties that proved to be of interest both within the theory itself and for its applications, such as homological properties and polynomial identities. Commutative rings are much better understood than non-commutative ones. Algebraic geometry and algebraic number theory, which provide many natural examples of commutative rings, have driven much of the development of commutative ring theory, which is now, under the name of commutative algebra, a major area of modern mathematics

Course Contents

Radical classes; Semisimple Classes; the Upper radical; Semis images; the lower radical; Heretariness of the lower radical class and the simple Upper radical class; Partitions of simple rings; Minimal left ideals; wedderbusn-Artin Structure theorem; the
Brown –McCoy radical; the Jacobson Radical; Connections among radical Classes: Homomorphically closed semisimple classes.

RECOMMENDED BOOKS

1. P. Hiegant, Radical and Semisimple Classes of Ring, Queen paper in pure and applied Mathematics, Queens University Kingston, Ontario, 1974.

MATH-708 ADVANCED TOPOLOGY

Credit Hours: 03+0

Objectives and Outcomes

Topology is one of the most active and advanced fields of mathematics, and it is indispensible for many other fields, such as Analysis, Geometry or Algebra. This course is an advance course of Topology.

Course contents

Directed sets and nets, subnets and cluster points, sequences and subsequences, quotient spaces, the Tychonoff theorem, completely regular spaces, the Stone-Eeihcompactification, meterization theorems and paracompactness, function spaces.

RECOMMENDED BOOKS:

1. J. L. Kelley, General Topology, Springer-Verlag, 1975.
2. S. Willard, General Topology, Addison Wesley Pub. Co., 1970.
3. J. R. Munkers, Topology (a first course), Prentice Hill Inc., 1975.

MATH-709 ALGEBRAIC TOPOLOGY

Credit Hours: 03+0

Objectives and Outcomes

This course is an introduction to algebraic topology. Algebraic topology studies topological spaces by associating to them algebraic invariants. The principal algebraic invariants considered in this course are the fundamental group (also known as the first homotopy group) and the homology groups. It is fundamental for students interested in research in Algebraic Geometry, Differential Geometry, Mathematical Physics, and Topology; it is also important for students in Algebra and in Number Theory.

Course contents

Homology theory: Homology groups, simplical homology, exact sequences. Singular homology.Cohomology, Duality and Topological manifolds.The Alexander Poincar's duality theorem.Generalhomotopy theory: some geometric construction. Homotopy classes of maps, Exact sequences, Fibre and cofibre maps.

RECOMMENDED BOOKS:

1. E. H. Spanier, Algebraic Topology, Tata McGraw Hill.
2. C. Kosniowski, A First Course in Algebraic Topology, Cambridge Uni. Press, 1988.
3. C. R. E. Maunder, Algebraic Topology, Cambridge Uni. Press,1980.
4. J. Mayer, Algebraic Topology, Prentice Hall NJ.

MATH-710 ADVANCED ANALYTICAL DYNAMICS-I
Credit Hours: 03+0

Objectives and Outcomes

This course is designed to teach the students how mathematics could be used in solving problems in the contemporary Science/Technology and Engineering world. Therefore, the course is structured to expose the students to the skills required to attain a level of proficiency in Analytical Dynamics.

Course contents

Equations of dynamics and its various forms; Equations of Lagrange and Euler, Jacobi's elliptic functions and the qualitative and quantitative Solutions of the problems of Euler and Poisson.The Problems of Lagrange and Poisson.Dynamical system; Equations of Hamilton and Appell; Hamilton–Jacobi theorem; Separable systems' Holder's variational principles and its consequences.

RECOMMENDED BOOKS

1. L. A. Pars, A Treatise on Analytical Dynamics, Heinman London, 1965.
2. E. T. Whittaker, A treatise on Dynamics of Rigid Bodies and Particles, Cambridge Uni. Press, 1965.

MATH-711 ADVANCE ANALYTICAL DYNAMICS-II
Credit Hours: 03+0

Objectives and Outcomes

The aim of this course is to give students advance topics analytical dynamics.

Course contents

Groups of continuous Transformations and Poincare's Equations; Systems with one degree of freedom; Singular Points; Cycle characteristics of systems with a Degree of freedom; Ergodic theorem; Metric indecompossability; stability of motion periodic Orbits.

RECOMMENDED BOOKS

1. L. A. Pars, A Treatise on Analytical Dynamics, Heinman London, 1965.
2. E. T. Whittaker, A treatise on Dynamics of Rigid Bodies and Particles, Camb, Uni.Press, 1965.

MATH-712 APPLIED FUNCTIONAL ANALYSIS
Credit Hours: 03+0

Objectives and Outcomes

The course gives an introduction to functional analysis and its applications, that constitute a fundamental tool in several central areas of mathematics and applied mathematics as, ordinary and partial differential equations, mathematical statistics and numerical analysis. This will be illustrated on some simplified practical applications coming from various engineering disciplines. The fundamental concepts of the operator and spectral theory will be explained and applied to diverse problems, for construction of approximations, convergence study and error estimates.

Course contents

Applications to bounded linear functional, Application to submmability of sequences, Numerical Integration and weak* convergence, Banach fixed point theorem and its applications to linear equations, differential equations and integral equations, Unbounded linear operators in quantum mechanics.

RECOMMENDED BOOKS

1. F. Riesz and Nagy, Functional Analysis, Frederick Ungar Publishing Co. 1965
2. E. Kreyszig "Introductory Functional Analysis with Applications", John, Wiley and Sons, New York. 1989
3. A. E. Taylor "Introduction to Functional Analysis", Wiley International Edition, New York. 1957

MATH-713 BANACH ALGEBRA
Credit Hours: 03+0

Objectives and Outcomes

Banach algebras have a lot of structure, combining the topological features of a Banach space, with the algebraic features of a ring. The main focus will be on examining Banach algebras consisting of continuous linear operators on Hilbert and Banach spaces.

Course contents

Banach Algebras; Ideals; Homomorphisms; Quotients Algebras; Winner's lemma; Gelfand's Theory of CommulativeBanach Algebras; The Notions of Gelfand's topology; Radicals; Gelfand's Transforms; Basic properties of Spectra;

Gelfand–Mazer Theorem; Symbolic Calculus; Differentiation; Analytic Functions; Integration of A–valued Functions; Normed Rings; Gelfand–Naimark Theorem.

RECOMMENDED BOOKS

1. W. Rudin, Functional Analysis, McGraw Hill Pub., N.Y.
2. M. A. Naimark, Normed Algebras, Wolters Noordhoff Pub., Netherland, 1972.
3. W. Zelazko, Banach Algebras, American Elsevier Pub., N.Y., 1973.
4. C. E. Rickart, Banach Algebras, D. Van Nostrand Pub., New York, 1960.

MATH-714 BIO-MATHEMATICS
Credit Hours: 03+0

Objectives and Outcomes

This course is intended to equip students with skills and techniques of model formulating, analysing and interpreting mathematical models in Biology, Ecology, Epidemics, etc.

Course contents

Biomathematics, Population, Population Density, Intrinsic growth rate, Logistic growth rate, Carrying capacity, Exponential growth, Incidence rate, Incidence density rate, Prevalence, Bilinear incidence rate, saturated incidence rate, epidemic, endemic, pandemic, modeling process, probability and rates, model classes, system of equations, reaction kinetics, A General Interaction Model for Two Populations, A Basic Epidemic Model, Non-dimensionalization.

Qualitative Analysis of 2 x 2 Systems: Phase-Plane Analysis: Linear Systems, Nonlinear Systems and Linearization, Qualitative Analysis of the General Population Interaction Model, Qualitative Analysis of the Epidemic Model, General Systems of Three or More Equations. Elementary Bifurcations: Saddle-Node Bifurcation, Trans critical Bifurcation, Pitchfork Bifurcation, Hopf Bifurcation, The Spruce Budworm Model. Continuous Population Models for Single Species: Continuous Growth Models, Insect Outbreak Model: Spruce Budworm. Models for Interacting Populations: Predator–Prey Models: Lotka–Volterra Systems, Complexity and Stability, Realistic Predator–Prey Models, Analysis of a Predator–Prey Model with Limit Cycle Periodic Behavior, Parameter Domains of Stability, Competition Models: Competitive Exclusion Principle. Routh Hurtwiz criteria, Local Stability, Positive definite functions.

RECOMMENDED BOOKS

1. A. J. Lotka, Elements of Mathematical Biology, Dover Publications, N.Y., 1956.
2. J.D. Murray, Mathematical Biology: I. An Introduction, Third Edition, Springer 2002.
3. G. D. Vries, T. Hillen, M. Lewis, J. Muller, S. Birgitt. A Course in Mathematical Biology, 2002.
4. Linda J.S. Allen, An Introduction to Mathematical Biology, 2006.

MATH-715 COMPUTATIONAL FLUID DYNAMICS
Credit Hours: 03+0

Objectives and Outcomes
Computational Fluid Dynamics (CFD), an extremely versatile technology requiring high performance computing environment is poised to take over as a universal software for simulating multi-physics problems in industrial R&D. The course is designed to reflect the broad range of CFD applications by providing a range of optional modules to address specific application areas.

Course contents
Prerequisites: Numerical Methods/Fluid Dynamics at M.Sc Level Philosophy of Computational Fluid Dynamics, Basic of Computational Fluid Dynamics: Incompressible plane flows, Stream function and vorticity equations, Conservative form and normalizing systems, Method for solving vorticity transport equation, Basic finite difference forms, Conservative property, Convergence and stability analysis, Explicit and implicit methods, Stream function equation and boundary conditions, Schemes for adjective diffusion equation, Upwind differencing and artificial vorticity, Solution for primitive variables.

RECOMMENDED BOOKS

1. C. A. J. Fletcher, Computational Techniques for Fluid Dynamics, Volume 1 & 2, Springer Verlag, 1992.
2. C. Y. Chow, Introduction to Computational Fluid Dynamics, John Wiley, 1979.
3. M. Holt, Numerical Methods in Fluid Mechanics, Springer Verlag, 1977.
4. H. J. Wirz and J. J. Smolderen, Numerical Methods in Fluid Dynamics, Hemisphere, 1978.
5. D. A. Anderson, J. C. Tannehill and R. H. Pletcher, Computational Fluid Dynamics and Heat Transfer, McGraw Hill, 1984.
6. J. D. Anderson, Computational Fluid Dynamics: The Basics with Applications, McGraw- Hill, 1995.
7. K. Hoffmann and S. T. Chiang, Computational Fluid Dynamics for Engineers, Vols. 1 and 2, Engineering Education System, 1993.

MATH-716 COMPUTATIONAL METHODS
Credit Hours: 03+0

Objectives and Outcomes
A study and analysis of important numerical and computational methods for solving engineering and scientific problems. The course will include methods for solving linear and nonlinear equations, evaluating integrals, solving ordinary and partial differential

equations, and determining eigenvalues and eigenvectors or matrices. The student will be required to write and run computer programs.

Course contents
Review of Numerical Methods (Bisection, Newton and Fixed point iteration), Householder method and Halley's method. Convergence of these methods. Review of Direct Methods for Linear Equations (Gauss, LUD, Tridiagonal), Cholesky Decomposition. Review of Matrix Algebra, Vector and Matrix Norms. Iterative Methods for solving linear equations (Jacobi, Gauss-Siedel, SOR methods), Eigen-value problems, Power method, Inverse power method and other techniques, Newton's method for system of non-linear equations, Numerical solution of Boundary Value Problems, Numerical solution of Partial differential equations.

RECOMMENDED BOOKS

1. R. L. Burden, J. D. Fairs;An Introduction to Numerical Analysis, 1993.
2. G. D. Smith, Numerical Solutions of P.D.Es, 1999.
3. J. H. Wilkinson, The Algebraic Eigenvlaue Problems, 1965.
4. U. Asher et al., Numerical solution of Boundary Value Problems in ODE's, 1986.

MATH-717 CONVOLUTION IN GEOMETRIC FUNCTION THEORY
Credit Hours: 03+0

Course Contents
The duality principle, test sets, convolution invariance Application to Geometric Function Theory, univalence criteria via convolution and applications prestarlike functions, applications to close-to-convex and related functions.

RECOMMENDED BOOKS

1. Convolutions and Geometric Function Theory by Ruscheweyh (1982)
2. Univalent Functions" by Ch. Pommerenke (1975)

MATH-718 DIFFERENTIAL SUBORDINATION THEORY
Credit Hours: 03+0

Course Contents
Subordination, Hypergeometic Functions, classes of functions, Integral operators, Differential operator. Second order differential subordination some fundamental results. The open Door Lemma and Integral Existence theorem.Classical subordination.

RECOMMENDED BOOKS

1. “Differential Subordinations” (2000) by S. S. Miller and P. T. Mocanu.
2. “Univalent Functions” Vol I & II, by A. W. Goodman

MATH-719 GEOMETRIC FUNCTION THEORY
Credit Hours: 03+0

Course Contents
Riemann mapping theorem, conformal mappings and their properties, univalent functions and their subclasses, Functions with positive real part, Herglotz Formula, Some basic properties of univalent and multivalent functions.

RECOMMENDED BOOKS

1. Geometric function theory and non-linear analysis by Tadeusz Iwaniec, Gaven Martin.
2. Topics in geometric function theory By Carl Hanson FitzGerald.
3. A. W. Goodman, Univalent Functions, Vol I & II.

MATH-720 INTEGRAL EQUATIONS
Credit Hours: 03+0

Objectives and Outcomes
This course unit consists of methods of solving various mathematical problems which arise in science. The method of Green's functions is a powerful tool in solving linear ordinary and partial differential equations, and the course starts with an introduction to this topic. There are situations where physical laws are better expressed as integral equations. On successful completion of this course students will be able to solve ordinary and partial differential equations.

Course contents
Existence Theorem; Integral Equations with L Kernels; Applications to Partial Differential Equations; Integral Transforms; Wiener-Hopf Techniques

RECOMMENDED BOOKS
1. H. H. Stad, Integral Equations, John Wiley, 1973.
2. I. Stakgold, Boundary Value Problems of Mathematical Physics, McMillan NY, 1968.

MATH-721 LARGE SCALE SCIENTIFIC COMPUTATION
Credit Hours: 03+0

Objectives and Outcomes

The overall goal of the course is to provide a basic understanding of how to develop algorithms and how to implement them in distributed memory computers using the message-passing paradigm and some advance methods in numerical computations. The students will be prepared either for research in an area where computational techniques play a significant role, or for a career in business or industry.

Course contents

Prerequisites: Scientific Computing, Numerical Analysis, Numerical Linear Algebra

Large sparse linear systems, Storage schemes, Review of stationary iterative process, Krylov subspace methods, Conjugate gradients(CG), BiCG, MINRES and GMRES, The Lanczos iteration, From Lanczos to Gauss quadrature, Preconditioning, Error bounds for CG and GMRES, Effects of finite precision

arithmetic, Multigrid methods, Multigrid as a preconditioner for Krylov subspace methods. Nonlinear systems, Newton's method and some of its variants, Newton GMRES, Continuation methods, Conjugate direction method, Davidon-Fletcher-Powell Algorithms.

Software Support: HOMPACK, LAPACK

RECOMMENDED BOOKS:

1. J. M. Ortega and W. C. Rheinboldt, Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, 1970.
2. C. T. Kelly, Iterative Methods for Linear and Nonlinear Equations, SIAM, Philadelphia, 1995.
3. A. Greenbaum, Iterative Methods for Solving Linear Systems, SIAM, Philadelphia, 1997
4. O. Axelsson, Iterative Solution Methods, Cambridge University Press, 1994.
5. P. Wesseling, An Introduction to Multigrid Methods, John Wiley & Sons, 1992.
6. C. W. Ueberrhuber, Numerical Computation: Methods, Software and Analysis, Springer Verlag, 1997.

MATH-722 ADVANCE MATHEMATICAL METHODS

Credit Hours: 03+0

Objectives and Outcomes

The course will cover some mathematical techniques commonly used in theoretical physics. This is not a course in pure mathematics, but rather on the application of mathematics to problems of interest in the physical sciences.

The students will learn the following topics:

- Hilbert spaces: complete orthonormal sets of functions
- Special functions (Legendre polynomials, Fourier series and integrals, spherical harmonics)
- Sturm-Liouville systems: orthogonal polynomials
- Green’s functions

Course contents

General solution of Bessel equation, Recurrence relations, Orthogonality of Bessel functions, Modified Bessel functions, Applications. General solution of Legendre equation, Legendre polynomials, Associated Legendre polynomials, Rodrigues

formula, Orthogonality of Legendre polynomials, Application. Concept and calculation of Green's function, Approximate Green's function, Green's function method for differential equations, Fourier Series, Generalized Fourier series, Fourier Cosine series, Fourier Sine series, Fourier integrals. Fourier transform, Laplace transform, Z-transform, Hankel transform, Mellin transform. Solution of differential equation by Laplace and Fourier transform methods.

RECOMMENDED BOOKS

1. G. N. Watson, A Treatise on the Theory of Bessel Functions, Cambridge University Press, 1944.
2. G. F. Roach, Green's Functions, Cambridge University Press, 1995.
3. A. D. Poularikas, The Transforms and Applications – Handbook, CRC Press, 1996.
4. J. W. Brown and R. Churchill, Fourier Series and Boundary Value Problems, McGraw Hill, 1993.

MATH-723 MATHEMATICAL TECHNIQUES FOR BOUNDARY VALUE PROBLEMS

Credit Hours: 03+0

Objectives and Outcomes

This course has a major focus on training analytical and logical thinking and learning fundamental methods for solving ordinary and partial differential equations. Both the knowledge about differential equations as well as the training of analytical faculties will be useful for the students in the course of their further studies. The course also explores the capacity and motivation for intellectual development through the solution of both simple and more complex mathematical problems from the important field of differential equations.

Course contents

Green's function method with applications to wave-propagation. Perturbation method: regular and singular perturbation techniques with applications. Variational methods.A survey of transform techniques; Wiener-Hopf technique with applications to diffraction problems.

RECOMMENDED BOOKS

1. A. Nayfeh, Perturbation methods.
2. I. Stakgold, Boundary Value Problems of Mathematical Physics.
3. B. Noble, Methods based on the Wiener-Hopf technique for the solution of Partial Differential Equations.
4. R. Mitra, S. W. Lee, Analytical Techniques in the Theory of Guided Waves.

MATH-724 MULTIVARIATE ANALYSIS-I
Credit Hours: 03+0

Objectives and Outcomes
The course is designed to understand the statistical analysis of the data collected on more than one (response) variable. These variables may be correlated with each other, and their statistical dependence is often taken into account when analyzing such data. This consideration of statistical dependence makes multivariate analysis somewhat different in approach and considerably more complex than the corresponding univariate analysis, when there is only one response variable under consideration.

Course contents
Introduction: Some multivariate problems and techniques. The data matrix Summary statistics. Normal distribution theory: characterization and properties linear forms. The Wish art distribution.The hostelling T^2 distribution. Distribution related to the multinomial. Estimation and Hypothesis testing.Maximum likelihood estimation and other techniques.The Behrens Fisher problem.Simultaneous confidence intervals.Multivariate hypothesis testing design matrices of degenerate rank. Multiple correlation. Least squares estimation discarding of variables.

RECOMMENDED BOOKS
1. K. V. Mardia, J. T. Kent, J. M. Bibby, Multivariate Analysis, Academic Press London, 1982.
2. A. M. Kshirsagar, Multivariate Analysis, Marcell Dekker, New York, 1972.

MATH-725 MULTIVARIATE ANALYSIS-II
Credit Hours: 03+0

Objectives and Outcomes
This is a course in multivariate statistical analysis, for students interested in quantitative methods of marketing research and more generally, for students of sciences. The aim of the course is to explore multivariate techniques used in modern marketing practice and in wider social research. Emphasis will be placed on case studies of marketing practice and on the practical application of the methods discussed. Topics to be drawn from: analysis of variance; regression analysis; principal components analysis; discriminant analysis; canonical correlation analysis; factor analysis; cluster analysis; multi-dimensional scaling.

Course contents
Principal component analysis: Definition and properties of principal comp .Hypotheses about principal components.Correspondenceanalysis.Discarding of variables.Principal component analysis in regression.Factoranalysis.The factor model.Relationships between factor analysis and principal component analysis.Canonical correlation analysis, dummy variables and qualitative data.Qualitative and quantitative data. Discriminant analysis: discrimination when the populations are known. Fisher’s linear discriminant function.Discrimination under estimation. Multivariate analysis of variance:

formulation of multivariate one-way classification. Testing fixed contrasis.Canonicalvariables and lest of dimensionality.Two-way classification.

RECOMMENDED BOOKS

1. K. V. Mardia, J. T. Kent, J. M. Bibby, Multivariate Analysis, Academic Press London, 1982.
2. A. M. Kshirsagar, Multivariate Analysis, Marcell Dekker, New York, 1972.

MATH-726 NUMERICAL ANALYSIS OF SPECTRAL METHODS

Credit Hours: 03+0

Objectives and Outcomes

This course is a mathematical introduction to appoximation theory, with a focus on spectral methods. The emphasis will be on both the analysis and the implementation of these methods. At the heart of both these methods is the same idea- the approximation of the solution by a (truncated) series expansion. The student will learn some basic theoretical results on spectral approximations for the issues of stability and convergence, on practical algorithms for implementing spectral methods, and on designing efficient and accurate spectral algorithms for solving PDEs of current interest.

Course contents

Introduction: Spectral Method: Survey of Approximation; Theory; Review of Convergence Theory; Algebraic Stability; Spectral Methods Using Fourier Series; Applications of Algebraic – Stability Analysis; Constant Coefficient Hyperbolic Equations; Time Differencing Efficient Implementations of Spectral Methods; Numerical Results of Hyperbolic Problems.

RECOMMENDED BOOKS

1. D. Gottlies, S. A. Orszag, Numerical Analysis of Spectral Method (Theory an Applications) J. W. Arrowsmith Ltd., England.
2. C. Canunto, M. Y. Hussani, AQuarteroni, T. Zang, Spectral Method in Fluid Dynamics, Springer-Verlag, N.Y.

MATH-727 NUMERICAL LINEAR ALGEBRA

Credit Hours: 03+0

Objectives and Outcomes

Students will learn the basic and advance direct methods for solving system of linear equations and linear least square equations, matrix factorization methods, basic computer arithmetic and the concepts of conditioning and stability of a numerical method, numerical methods for computing eigenvalues and their derivation, basic iterative methods, singular value decomposition. They will also improve their problem solving skills in computational linear algebra.

Course contents

Matrix-Vector operations, Orthogonal vectors and matrices, Matrix and vector norms, Singular value decomposition (SVD), Projectors and QR factorization, Gram-Schmidt orthogonalization process, Householder triangularization, Least-squares problems, Condition numbers, Gaussian elimination and LU factorization, Pivoting and LUP factorization, Stability of Gaussian elimination, Cholesky Factorization, Overview of eigenvalue problems, Reduction to upper- Heisenberg Tridiagonal form, Power and inverse power iteration, QR algorithm without shifts, QR algorithm with shifts, Arnold iteration, GMRES method, Lanczos iteration Orthogonal polynomials and Gauss quadrature, Conjugate gradient (CG) method, Bi-Orthogonalization method.

RECOMMENDED BOOKS

1. L. N. Trefethen, D. Bau,Numerical linear algebra, SIAM, Philadelphia, 1997.
2. G. Allaire, S. M. Kaber, K. Trabelsi,Numerical Linear Algebra, Springer Science+Business Media, LLC, 2008.
3. W. Brandal, Numerical Linear Algebra.
4. L. Fox, An introduction to numerical linear algebra.

MATH-728 NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS-I
Credit Hours: 03+0

Objectives and Outcomes

The course deals with integral equations, their origin, properties and solutions, both approximate and numerical.

Course contents

INTRODUCTION TO THE THEORY OF INTEGRAL EQUATIONS:

Definition; Existence Theorems; Integral Equations with L Kernels; Quadrature and Fourier Series; Function Spaces and Linear Operators; Basic Approximation Theory.

QUADRATURE METHODS FOR FREDHOLM EQUATIONS OF THE SECOND KIND

Introduction; Formulation of Discrete Equation; Choice of Quadrature Formula; Use of Finite Differences; Deferred Approach to the Limit; Nonlinear Equations; Singular Integral Equations; Removal of the Diagonal Term; Use of Peduct Integration Methods; Singularity in the Solution; Error Analysis of Quadrature Methods

EXPANSION METHODS

Nature of Approximating Function; Criteria for Determining the Approximation; Choice of (x), & (x)…..; Theory of Projection Methods; Other methods:

RAYLEIGH-RITZ-GALERKIN METHODS

Introduction; The Eigenvalue Problem; Inhomogeneous Equations; Error Estimates; Numerical Performance; Extension to Nonlinear Equations; Comparison with Other Methods ;

RECOMMENDED BOOKS

1. C. T. H. Baker, Integral Equations, Cleradon Press, 1977.

2. F. Smithies, Integral Equations, Cambridge University Press, 1958.
3. Squire, Numerical Integration for Engineer, American Elsevier Publishing Co., 1970.

MATH-729 NUMERICALSOLUTIONS OF INTEGRAL EQUATIONS-II
Credit Hours: 03+0

Objectives and Outcomes
This course is an introduction to fast solvers for integral equations, the course will concentrate mainly on integral equations arising from elliptic problems but, if time permits, the parabolic and hyperbolic cases will be briefly outlined. At the end of the course the student will be able to comfortably apply various numerical techniques for the solution of different kinds of integral equations.

Course contents
NUMERICAL SOLUTIONS OF THE EIGENVALUE PROBLEM
Methods based on Quadrature Rules; Treatment of Discontinuities Using Methods Based on Approximate Integration; Expansion Methods for Eigenproblem

VOLTTERRA EQUATIONS OF THE SECOND KIND
Introduction; Multistep Methods; Runge-Kutta Methods; Bock methods; Spline Approximations; Convergence and Stability;

METHODS FOR VELTERRA EQUATIONS OF THE FIRST KIND
Introduction; Conversion to Equations of the Second Kind; Numerical Methods of Solution; Use of Quadrature Rules in the Equation of the First Kind; higher Order Accuracy; Product Integration

FREDHOLM EQUATIONS OF THE FIRST KIND
Introduction; nature of the Problem, Singular Function Analysis; Fundamental Theorems; Applications; noise; need for Filtering; Methods of Expansion in Singular Functions; Use of Other Expansions; Methods of Regularization; Interactive Methods

RECOMMENDED BOOKS:
1. C. T. H. Baker, Integral Equations, Cleradon Press, 1977.
2. F. Smithies, Integral Equations, Cambridge University Press, 1958.
3. Squire, Numerical Integration for Engineer, American Elsevier Publishing Co., 1970.

MATH-702 NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS
Credit Hours: 03+0

Objectives and Outcomes

This course will provide an overview of classical solution methods for ordinary equations. The focus will be on one step, multistep, finite–difference and shooting techniques and their stability and convergence. At the end of the course, the students will be able to implement several numerical techniques for finding approximate solutions to ordinary differential equations.

Course contents

PRELIMINARIES:

Some Theorems from the Theory of Differential Equations; Initial Value Problems for First Order Ordinary Differential Equations and for Systems of First Order Ordinary Differential Equations; Deduction of Higher Order Differential Equations to First-Order Linear System's with Constant Co-efficient; Linear Difference; Equations with Constant Co-efficient

LINEAR MULTISTEP METHODS:

The General Linear Multistep Methods; Derivation Through Taylor Expansions; Derivation Through Numerical Integration; Derivation Though Interpolation;

Convergence; Order and Error Constant; Local and Global Truncation Error; Consistency and Numerical Stability; Attainable order of Stable Methods.

Problems in Applying Linear Multistep Methods; Starting Values; A Bound for the Local Truncation Error; Weak Stability; General Methods for Finding Intervals of Absolute and Relative Stability; Predictor-Corrector Methods; The Local Truncation Error of Predictor-Corrector Methods; Weak Stability of Predictor-Corrector Methods. Introduction; Order and Convergence of the General Explicit One-Step Method; Derivation of Classical Runge-Kutta Methods; Runge-Kutta Methods of order Greater Than Four; Error Estimates and Error Bounds for RungeKutta Methods; Comparison with P Predictor-Corrector Methods; Implicit Runge-Kutta Methods.

RECOMMENDED BOOKS

1. Greenspan, Numerical solutions of ODE's for classical Relativistic and Nanosystems, 2006.
2. C. E. Froberg, Numerical mathematics, The Benjamin Cummings Pub. Com. Inc., 1985.
3. G. M. Phillips, P. J. Taylor, Theory and Applications of Numerical Analysis, Academic Press, 1973.
4. W. E. Pre et al., Numerical Recipes, Cambridge University Press, 1986.
5. M. K. Jain, Numerical Solution of Differential Equations, Wiley Eastern Ltd.
6. W. E. Milne, Numerical Solution of Differential Equations, Dover Pub. Inc., N.Y.

MATH-730 NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS
Credit Hours: 03+0

Objectives and Outcomes

This course explores an introduction to various numerical methods for partial differential equations with emphasis on finite difference type methods. The course also illustrates, via numerics, the distinguishing mathematical properties of various commonly occurring partial differential equations.

Course contents

Numerical Methods for Parabolic PDEs; review of finite difference methods, explicit methods, Crank-Nicolson implicit method, Local Truncation Error, Stability, Consistency and convergence, Fourier stability methods, alternating directions implicit method, higher level schemes, nonlinear equations, predictor corrector methods, computer problems,Two dimensional parabolic equations and finite difference schemes,computer problems. Numerical methods for hyperbolic PDEs; method of characteristics, finite-difference methods-Lax-Wendroffmethod, Courant-Friedrichs-Lewymethod, two-space Hyperbolic equations, computer implementations. Numerical Methods for Elliptic PDEs; finite-difference methods, Poisson Equation, Laplace Equations, Curved boundary, finite-differences in Polar co-ordinates.

RECOMMENDED BOOKS

1. C. Jhonson, Numerical Solutions of Partial Differential Equations by the finite methods, Cambridge University Press
2. W. F. Ames, Numerical methods for P.D.Es, Academic Press.
3. G. D. Smith, Numerical Solutions of P.D.Es finite difference methods, Clarendon Press, Oxford.
4. G. W. Thomas, Numerical Solutions of P.D.E's.

MATH-731 OPTIMIZATION THEORY-I

Credit Hours: 03+0

Objectives and Outcomes

This course deals with the mathematical theory of optimization. Theory and algorithms for nonlinear optimization, focusing on optimization. Quasi-Newton methods; conjugate-gradient and methods for large-scale problems; algorithms for least-squares problems linear and nonlinear equations; constrained optimization. Upon successful completion of this course, the student will be able to understand:

1. Basic theoretical principles in optimization;
2. Formulation of optimization models;
3. Solution methods in optimization.

Course contents

Statement of the problem, condition for optimality, concept of direction of search, alternating direction and steepest descent methods, conjugate direction method, conjugate gradient method, Newton's method, Quasi-Newton equation, derivation of updating formulae for Quasi-Newton equation, The Gauss-Newton method, The Levenberg-Marquart method, The corrected Gauss-Newton method, Methods for large scale problems.

RECOMMENDED BOOKS:

1. P. E. Gill, E. Murray, H. H. Wright, Practical Optimization, Academic Press, 1981.
2. R. Fletcher, Practical Methods of Optimization Vol.I& II, John Wiley and Sons, 1980.
3. S. S. Rao, Optimization Theory and Application, Wiley Eastern Ltd., 1984.
4. D. G. Luenberger, Optimization by Vector Space Methods, John Wiley & Sons, 1968.
4. D. G. Luenberger, Introduction to Linear & Nonlinear Programming. Addison Wesley Publishing Co. Sydney, 1965.
5. M. S. Bazara, C. M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley, 1979.

MATH-732 OPTIMIZATION THEORY-II

Credit Hours: 03+0

Objectives and Outcomes

This course deals with theory and algorithms for nonlinear optimization, focusing on unconstrained optimization. Algorithms for problems and nonlinear equations; gradient projection algorithms for bound-constrained problems; Lagrange's multiplier methods for nonlinearly constrained optimization.

Course contents

Theory of constrained optimization, condition of optimality, methods for minimizing a general function subject to linear equality constraints, active set strategies for linear inequality constraints, special forms of the objectives functions, Lagrange multiplier estimates, Changes in working set, Barries function methods, Penalty functions methods, Methods based on Lagrangian functions reduced gradient and gradient projection methods.

RECOMMENDED BOOKS:

1. P. E. Gill, E. Murray, H. H. Wright, Practical Optimization, Academic Press, 1981.
2. R. Fletcher, Practical Methods of Optimization Vol.I& II, John Wiley and Sons, 1980.
3. S. S. Rao, Optimization Theory and Application, Wiley Eastern Ltd., 1984.

4. D. G. Luenberger, Optimization by Vector Space Methods, John Wiley & Sons, 1968.
5. D. G. Luenberger, Introduction to Linear & Nonlinear Programming. Addison Wesley Publishing Co. Sydney, 1965.
6. M. S. Bazara, C. M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley, 1979.

MATH-733 PARTIAL DIFFERENTIAL EQUATIONS
Credit Hours: 03+0

Objectives and Outcomes
This course is an advance to the theory of partial differential equations (PDEs). Upon successful completion the student will learn linear operators and linearity, partial differential equation and associated boundary and initial value problems, and well posed problems, the concept of maximum principle, existence and uniqueness, the concept of Green's functions and be able to derive them and use them in some simple cases, the notions of Poisson's equation and the Poisson integral formula, three-dimensional wave equation.

Course contents
Cauchy's Problems for Linear Second Order Equations in n-independent Variables; Cauchy Kowalewski Theorem; Characteristic Surfaces; Adjoint Operators; Bicharacteristics; Spherical and Cylindrical Waves; Heat Equation; Wave Equation; Laplace Equation; Maximum-Minimum Principle; Integral Transforms.

RECOMMENDED BOOKS
1. R. Dennemyer, Introduction to P.D.E's & Boundary Value Problems, McGraw Hill Com., 1968
2. C. R. Chester, Techniques in P.D.E's, McGraw Hill Com., 1971.

MATH-734 PERTURBATION METHODS
Credit Hours: 03+0

Objectives and Outcomes
The aim of the course is to lay an introduction to the perturbation theory to solve ordinary differential equations, partial differential equations as well as integral equations. The emphasis in this course is on the adaptation of several mathematical methods and techniques to their swift and effective application in solving advanced problems in applied mathematics and theoretical physics.

Course contents

Difference equations, Dimensional analysis, Expansions, Approximate solutions of linear differential equations, order symbols, Asymptotic series, Quadratic and cubic algebraic equations and its solutions by perturbation method, Straightforward expansion, Lindsted-Poincar Technique, Method of Renormalization, Method of multiple scales, dominant balance method, WKB method, Method of strained parameters.

RECOMMENDED BOOKS

1. Alan W. Bush, Perturbation methods for engineers and scientists,CRC Press.
2. C. Bender, S. Orszag, Advanced mathematical methods for scientists and engineers,MGH, 1978.
3. E. Zauderer, Partial Differential Equations of Applied Mathematics, T 2nd edition ,1998.
4. Ali Hasan Nayfeh, Introduction to perturbation techniques, Awiley-Interscience Publication, John Wiley & SONS, INC.

MATH-735 ADVANCED PERTURBATION METHODS
Credit Hours: 03+0

Objectives and Outcomes

The goal of the course is to study some advance method to the perturbation theory to solve ordinary differential equations, partial differential equations as well as integral equations.

Course contents

Regular perturbation, Singular perturbation, Boundary layer, The method of matched asymptotic expansion, equations with large parameter, , Solution of partial differential equations by perturbation methods, Asymptotic expansion of integrals Laplace's method, Watson's Lemma, Riemann-Lebesgue lemma.

RECOMMENDED BOOKS

1. Alan W. Bush, Perturbation methods for engineers and scientists, CRC Press.
2. C. Bender, S. Orszag, Advanced mathematical methods for scientists and engineers, MGH, 1978.
3. E. Zauderer, Partial Differential Equations of Applied Mathematics, T 2nd edition, 1998.
4. Ali Hasan Nayfeh, Introduction to perturbation techniques, Awiley-Interscience Publication, John Wiley & SONS, INC.

MATH-736 PROBABILITY AND PROBABILITY DISTRIBUTIONS-I
Credit Hours: 03+0

Objectives and Outcomes

The objective of the course is to study some important topics of probability theory. It focuses on probability distributions and estimation methods.

Course contents

Historical origin of term Probability, Conditional Probability, Baye'stheorem, Chebyshev inequality. Random variables, Distribution function, Probability density function, Probability distribution of two variables, Binomial, Poisson, Hyper geometric, Negative Binomial, Geometric, Uniform, Exponential, Beta, Gamma and Normal distributions, Bivariate Normal distribution, Multivariate normal distribution, Central, limit Theorem, Probability as based of estimation, Properties of good estimator, Unbaised, Consistent, Sufficient, Efficient estimators, Minimum variance unbiased estimators,

RECOMMENDED BOOKS

1. A. Stuarts, Ord, J. K. Kendalls, Advanced theory of mathematics (Vol I) CharlesCoriffi& Co, London.
2. A. M. Mood, Graybill, D.C.Boes, Introduction to the theory of statistics, McGraw Hill, NY.
3. R. M. Hogg, A. T. Craig, Introduction to mathematical statistics, McMillan Co., New York.
4. A. S. Hirai, Estimation of statistical parameters, IlmiKitabKhana, Lahore, Pakistan.
5. R. E. Walpole, Introduction to mathematical statistics.

MATH-737 PROBABILITY AND PROBABILITY DISTRIBUTIONS-II
Credit Hours: 03+0

Objectives and Outcomes

This course introduces the basic notions of probability theory and develops them to the stage where one can apply the probabilistic ideas in statistical inference and modeling, and the study of stochastic processes.

Course contents

Moments generating function and characteristic functions, Cauchy distribution, Laplace distribution, Weiball distribution, Maxwell distribution, Pareto distribution, Raleigh distribution, Lag normal distribution, Inversion and uniqueness theorems, Convolution of function, Sampling distribution, Distribution of mean, median, rang and quartiles, Central and Non Central t, F and F and Chi-Square distribution, Neyman Pearson theorem, Uniform most powerful tests, like hood ratio tests, The sequential probability ratio test, Interval estimation for different parameters.

RECOMMENDED BOOKS

1. A. Stuarts, Ord, J. K. Kendalls, Advanced theory of mathematics (Vol I), Charles Coriffi& Co, London.
2. A. M. Mood, Graybill, D.C.Boes, Introduction to the theory of statistics, McGraw Hill, NY.
3. R. M. Hogg, A.T.Craig, Introduction to mathematical statistics, McMillan Co., New York.
4. A. S. Hirai, Estimation of statistical parameters, IlmiKitabKhana, Lahore, Pakistan.
5. R. E. Walpole, Introduction to mathematical statistics.

MATH-738 SEMI GROUP THEORY
Credit Hours: 03+0

Objectives and Outcomes

The aim of the course is to familiar the student with the concepts of semi group theory. The formal study of semi groups began in the early 20th century. Semi groups are important in many areas of mathematics because they are the abstract algebraic underpinning of "memoryless" systems: time-dependent systems that start from scratch at each iteration. In applied mathematics, semi groups are fundamental models for linear time-invariant systems. In partial differential equations, a semi group is associated to any equation whose spatial evolution is independent of time. The theory of finite semi groups has been of particular importance in theoretical computer science since the 1950s because of the natural link between finite semi groups and finite automata. In probability theory, semi groups are associated with Markov Process.

Course contents

Introductory Ideas; Basic Definitions; Cyclic Semi groups; Order Sets; Semi Lattices and lattices; Binary Relations; Equivalences; Congruence; Free Semi groups; Green's Equivalences; L,R,H,J and D, Regular Semi groups; O–Simple Semi groups; Simple and O–Simple Semi groups; Rees's Theorem; Primitive Idempotent; Completely O–Simple Semi groups; Finite Congruence–Free Semi groups; Union of Groups; Bands; Free Bands; Varieties of Bands; Inverse Semi groups; Congruence on Inverse Semi groups; Fundamental Inverse Semi groups; Bisimple and Simple Inverse Semi groups; Orthodox Semi groups.

RECOMMENDED BOOKS

1. A. H. Clifford, G. B. Preston, The Algebraic Theory of Semigroups Vol. I & II, AMS Math, Survey, 1961 & 1967
2. J. M. Houie, An Introduction to Semigroups Theory, Academic Press, 1967.

MATH-739 VISCOUS FLUID-I
Credit Hours: 03+0

Objectives and Outcomes

This course explores viscous fluid and its applications. The course material can be used as a reference source for future real world situations. After successful completion of this course, students will be able to understand the concept and solve viscous fluid problems.

Course contents

Eulerian approach, Lagrangian description, Properties of fluids, Transport properties, Kinematic properties, thermodynamics properties, Boundary conditions for viscous flows and heat conducting flows problems, Conservation of mass (equation of continuity), conservation of momentum (equations of Navier-Stokes), conservation of energy (energy equations), Dimensionalization and dimensionless parameters in viscous flow, Vorticity transport equation, Stream function, Steady flow, unsteady

flow, creeping flow and boundary layer flow, Couette flows, Poiseuille flow, CouettePoiseuille flow between parallel plates, Stokes first problem, Stokes second problem,

Unsteady flow between two infinite plates, Asymptotic suction flows: uniform suction on a plane, flow between parallel plates with top suction and bottom injection.

RECOMMENDED BOOKS

1. Frank M. White ,Viscous Fluid Flow, Second Edition, McGRAW-HILL, Inc.
2. Hermann Schlichting,Boundary-layer Theory ,Seventh Edition, McGraw-Hill Series in Mechanical Engineering.
3. G.K. Batchelor, An introduction to fluid dynamics, Cambridge University Press.

MATH-740 VISCOUS FLUID-II
Credit Hours: 03+0

Objectives and Outcomes

This course deals with the advance methods for the solution of viscous fluid problems. After successful completion of this course, students will be able to solve viscous fluid problems in higher dimensions.

Course contents

Similarity solution, Berman problem, Plane stagnation flow, axisymmetric stagnation flow, flow near an infinite rotating disk, Jeffery Hammel flow in a wedge shaped region and it solution for small wedge angle, Stokes solution for an immersed sphere, Derivation of boundary-layer equations for two-dimensional flow, The laminar boundary layer equations, The approximate method due to the von Karman and K. Pohlhausen for two dimensional flows, Blasius problem of flat plate flow, Falker-Skan wedge flows, Heat transfer for Falker-Skan flows, two dimensional steady free convection, viscous flows over a stretching sheet, thin film flows

RECOMMENDED BOOKS

1. Frank M. White ,Viscous Fluid Flow, Second Edition, McGRAW-HILL, Inc.
2. Hermann Schlichting, Boundary-layer Theory ,Seventh Edition, McGraw-Hill Series in Mechanical Engineering.
3. G. K. Batchelor, An introduction to fluid dynamics, Cambridge University Press.

MATH-741 THEORY OF AG-GROUPOIDS
Credit Hour: 03+0

Course Outline:
Introduction, Basic subclasses of AG-groupoids:AG-monoids; AG*-groupoids AG**-groupoids; Cancellative AG-groupoids; AG-groups.New subclasses of AG-groupoids:
Quasi-cancellative AG-groupoids; Paramedial AG-groupoids; Alternative and flexible AG-groupoids; Self-dual AG-groupoids and unipotent AG-groupoids; Type1, Type2, Type 3 and Type 4 AG-groupoids.Enumeration and constructions of AG-groupoids:Enumeration and classifications of AG-groupoids; Constructions of AG-groupoid;Enumeration and constructions of AG-groups.A study of AG-group as generalization of abelian groups:Cosets and complexes in AG-group; Normality in AG-group; Commutators in AG-group; Direct product of AG-groups;AG-group action.
A study of AG-groups as quasigroups:Generalizations of several concepts of loops to AG-groups; Multiplication group of an AG-group;AG-groups as parallelogram spaces.

Recommended Books

1. A. Distler, M. Shah, and V. Sorge,Enumeration of ag-groupoids, LNCS, 2011, Volume 6824, 1-14. Springer Verlag.
2. Q. Iqbal and Q. Mushtaq, Decomposition of locally associative LA-semigroup, Semigroup forum, 41:155{164, 1990.
3. M. Khan, Some studies in AG**-groupoids, Phdthesis,Deparment of Mathematics, Quaid-i-AzamUniversity,Islamabad,Pakistan, 2008:
4. M. Shah, A Theoretical and Computational Investigation of AG-groups, PhD thesis, 2012.
5. M. Shah, T. Shah, and A. Ali On the cancellativity of AG-groupoids. IMF, 6(44):2187{2194, 2011.

MATH-742 VARIATIONAL INEQUALITIES
Credit Hours: 03+0

Objectives and Outcomes
The aim of the course is to introduce the student to the theory of varitional inequalities. The student will learn a number of standard and powerful algorithms, as well as demonstrating methodologies in variational inequalities as well as in optimization theory.

In addition the student will be introduced to the use of different tolls like Maple and Matlab.

Course Contents

Variational Problems, Existence results for the general implicit variational problems, Implicit Ky-Fan's inequality for monotone functions, Jartman Stampacchia theorem for monotone compact operators, Selection of fixed points by monotone functions, Variational and quasivariational inequalities for monotone operators.

RECOMMENDED BOOKS

1. J.L. Liions., and G. StamPacchia, Variational Inequalities, Comm. Pure Appl. Math 20, 1967.
2. V. Mosco., Implicit Variational Problems and Quasi Variational Inequalities, Lecture Notes in Mathematics-543, Springer-Verlag, Berlin, 1976.
3. C. Baiocchi and A. Capelo, Variational and Quasi-variational Inequalities, Wiley, 1984.

MATH-743 Fuzzy Algebra
Credit Hours: 03+0

Objectives and Outcomes

The Course of fuzzy sets provide the basic idea of fuzzy and uncertainty. By reading this course the students will able to apply the fuzzy concept in Algebra. While the generalized fuzzy sets will provide the basic operations of addition, subtraction, multiplication etc. in in the context of fuzzy sets.

Course Outlines

Intoduction to Fuzzy Algebra, The Concept of Fuzziness Examples, Mathematical Modeling, Operations of fuzzay sets, Fuzziness as uncertainly. Algebra of Fuzzy Sets Boolean Algebra and lattices, Equivalence relations and portions, Composing mappings, Alpha-cuts, Images of alpha-level sets, Operaions on fuzzy sets.
Fuzzy Relations: Definition and examples, Binary Fuzzy relaionsOperaions on Fuzzy relations, fuzzy partitions. Fuzzy ideals of semigroups, Fuzzy quasi-ideals, Fuzzy bi-ideals of Semigroups, Characterization of different classes of semigroups by the properties of their fuzzy ideals, fuzzy quasi-ideals and fuzzy bi-ideals.
Generalized fuzzy sets, generalized fuzzy ideals, generalized fuzzy bi-ideals and quasi-ideals, characterizations of semigroups using these concepts. Fuzzy ideals of rings, prime, semiprime fuzzy ideals, Characterization of rings using the properties of fuzzy ideals, Generalized fuzzy ideals and applications of these concepts in rings.

Recommended Books:

1. Hung T. and Nguyen, A First course in Fuzzy logic, Chapman and Hall/CRC. Elbert A. Walker 1999.

2. M. Ganesh, Introductioin to Fuzzy Sets and Fuzzy logic, Prentice-Hall of India, 2006.
3. John N. Mordeson, Fuzzy Commulative algebra, WolrdScienticic, 1998. D. S. Malik.
4. John N. Mordeson, Fuzzy Semigroups, Springer-Verlage, 2003. D.S. Malikand, Nobuki Kuroki.

MATH-744 General Relativity
Credit Hours: 03+0

Course outline:

Review of special Relativity, tensors and field theory. The principles of which General relativity is based. Einstein's field equation obtained from geodesic deviation. Vacuum equation.The schwarzchild exterior solution.Solution of the Einstein Maxwell field equations and the Schwarzchild interior solution.The Kerr-Newnann solution (without Derivation).Foliations, Relativistic corrections to Newtonian gravity. Black holes, the Kruskal and Penrose diagrams. The field theoratic derivation of Einstein's equations.Weak field approximations and gravitational waves.Isometries, conformal transformations, problems of quantam gravity.

Recommended Books:

1. A. Qadir, Relativity: An introduction to special theory, world scientific, 1989.
2. Misner C.W, Thorne, K.S, Wheeler J.A, Graviatation, Freeman 1974.
3. Hawking S.W and Ellis, G.F.R, The large scale structures of space time, Acadamic press, 1972.

MATH-746 ADVANCE FUNCTIONAL ANALYSIS
Credit Hours: 03+0

Objectives and Outcomes

The course combines ideas and methods from different areas of mathematics. It is designed especially for students who want to choose operator algebras as their speciality, but the content of the course will also be useful to students interested in other branches of analysis. This point of view turned out to be particularly useful for the study of differential and integral equations.

Course Contents

Normed Spaces.Banach spaces; Bounded Linear operators; Compactness and Continuity; Finite Dimensional Normed Linear spaces; The Stone Weirstrass Theorem and AseoliAezela Theorem; Bounded linear Functionals; Dual spaces; The Hahn Banach Theorem. The Riesz Representation Theorem. Contraction, Fixed Point Theorem and its applications, Reflexive spaces. Strong and Weak Convergence.Convergence of Sequences of Operators and Functionals; Bair's Theorem; The Principle of Uniform Boundedness; The Open Mapping Theorem and

Closed Graph Theorem; Compact Linear Operators, Applications in Approximation Theory.

RECOMMENDED BOOKS.

4. A. L. Brown, A. Page, Elements of Functional Analysis, Van Nostl found and ReinholtCompany London, 1997.
2. E. Kreyszig, Introductory Functional Analysis With Applications, John Wiley & Sons, N. Y.1989.
3. A. E. Taylor, D. C. Lay, Introduction to Functional Analysis, 2nd Edition. Robert E. Krieger Pub. Company Florida, 1986.
4. J. D. Conway, A Course in Functional Analysis, Spring-Verlag, 1994.

MATH-747 FIXED POINET THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Introduction to Fixed Point theory and its applications. Banach fixed point theorem. Contractive mapping. Kannan and Chatterjea, Zamfirescu fixed point theorems. Ciric's fixed point theorem. Weissinger, Rakotch, Boyd-Wong, Meir-Keeler, Hardy and Rogers and other generalization of contraction mapping. Common fixed point. Fixed point theorem for nonexpansive mappings and related classes of mappings. Quasi nonexpansive mappings. Fixed points of multivalued mappings.

RECOMMENDED BOOKS

1. K. Goebel and W.A. Kirk, Topics in Metric Fixed Point Theory
2. , Cambridge University Press, 1990.
3. J. Dugundji and A.Granas, Fixed Point Theory, Polish Scientific Publishers, Warszawa, 1982.
4. V.I. Istratescu, Fixed Point Tjepru. D. Reidel Publication Company, 1981.
5. K. Goebel and S. Reich, Uniform Convexity, Hyperbolic Geometry and Nonexpansive Mapping, Marcel Dekker Inc. 1984.

MATH-748 TOPICS IN COMPLEX ANALYSIS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Analytic continuation, equicontinuity and uniform boundedness, normal and compact families of analytic functions, external problems, harmonic functions and their properties, Green's and von Neumann functions and their applications, harmonic measure, conformal mapping and the Riemann mapping theorem, the kernel function, functions of several complex variables.

RECOMMENDED BOOKS

1. Nehari. Z., Conformal mappings, Constable and Company.
2. Hille, E., Analytic function theory, Chelsea.
3. Sansone, G., and Gewetsen, J., Lectures on the theory of function of a complex variable, Wolters-Noordhoff, Vol.II.

MATH-749 MATHEMATICAL MODELING-I
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Introduction to Modelling. Collection and interpretation of data. Setting up and developing models. Checking models. Consistency of models. Dimensional analysis. Discrete models. Multivariable models. Matrix models. Continuous models. Modelling rates of changes. Limiting models. Graphs of functions as models. Periodic models. Modelling with difference equations. Linear, Quadratic and Non-Linear Models.

RECOMMENDED BOOKS

1. Edwards, D. and Hamson, M.: Mathematical Modelling Skills (Macmillan Press Ltd., 1996).
2. Giordano, F.R., Weir, M.D. and Fox, W.P.: A First Course in Mathematical Modelling (Thomson Brooks/Cole, 2003).
3. Law, A.M. and Kelton, W.D.: Simulation Modelling and analysis (McGraw-Hill, 1982).
4. Spriet, J.A. and Vnsteenkiste, G.C.: Computer Aided Modelling and Simulation,(Academic Press, 1982).
5. Aris, R.: Mathematical Modelling Techniques (Dover Publication, 1995).

MATH-750 MATHEMATICAL MODELING-II

Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Modeling with Differential Equations: Exponential growth and decay. Linear, non-linear systems of differential equations. Modeling with integration. Modeling with random numbers: Simulating qualitative random variables. Simulating discrete random variables. Standard models. Monte Carlo simulation. Fitting models to data. Bilinear interpolation and Coons patch.

RECOMMENDED BOOKS

1. Edwards, D. and Hamson, M.: Mathematical Modelling Skills (Macmillan Press Ltd., 1996).
2. Giordano, F.R., Weir, M.D. and Fox, W.P.: A First Course in Mathematical Modelling (Thomson Brooks/Cole, 2003).
3. Law, A.M. and Kelton, W.D.: Simulation Modelling and analysis (McGraw- Hill, 1982).
4. Spriet, J.A. and Vnsteenkiste, G.C.: Computer Aided Modelling and Simulation, (Academic Press, 1982).

MATH-751 OPTIMAL CONTROL
Credit Hours: 03+0

Objectives and Outcomes

Course Contents
Basic Optimal Control Problems: Preliminaries, The Basic Problem and Necessary Conditions, Pontryagin's Maximum Principle. Existence and Other Solution Properties: Existence and Uniqueness Results, Interpretation of the Adjoint, Principle of Optimality, The Hamiltonian and Autonomous Problems, State Conditions at the Final Time, Payoff Terms, States with Fixed Endpoints, Bounded Controls, Necessary Conditions, Numerical Solutions, Optimal Control of Several Variables, Necessary Conditions, Linear Quadratic Regulator Problems, Higher Order Differential Equations, Isoperimetric Constraints, Numerical Solutions, Epidemic Model, HIV Treatment, Bear Populations, Glucose Model, Free Terminal Time Problems, Necessary Conditions, Time Optimal Control, Adapted Forward-Backward Sweep, Secant Method, One State with Fixed Endpoints, Nonlinear Payoff Terms, Free Terminal Time.
Partial Differential Equation Models: Existence of an Optimal Control, Sensitivities and Necessary Conditions, Uniqueness of the Optimal Control, Numerical Solutions, Harvesting Example, Beaver Example, Predator-Prey Example, Identification Example, Controlling Boundary Terms.

RECOMMENDED BOOKS

1. Suzanne Lenhart John T. Workman, Optimal Control Applied to Biological

Models, 2007 by Taylor & Francis Group, LLC

MATH-752 NEAR RINGS-1
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Near Rings, Ideals of Near-rings, Isomorphism Theorems, Near Rings on finite groups, Near-ring modules. Isomorphism theorem for R-modules, R-series of modules, Jorden-Holder- Schrier Theorem, Type of Representations, Primitive near-rings R-centralizers, Density theorem, Radicals of near-rings.

RECOMMENDED BOOKS

1. Pilz, G., Near Rings, North Holland.

MATH-753 NEAR RINGS-II
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Distributively generated near-rings, ideals isomorphism theorems, Free d.g. near rings, Representations of d.g. near-rings, Types of representations, upper and lower faithful d.g. near rings, Endomorphism near-rings of groups.

RECOMMENDED BOOKS

1. Pilz, G., Near-Rings, North Holland, 1976.

MATH-754 SEVERAL COMPLEX VARIABLE
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Holomorphic functions: Review of 1-variable theory, Real and complex differentiability, Power series, Complex differentiable functions, Cauchy integral formula for a polydisc, Cauchy inequalities, The maximum principle.

Extension of analytic functions: Hartogs figures, Hartogs theorem, Domains of holomorphy, Holomorphic convexity, Theorem of Cartan Thullen.
Levi-convexity: The Levi form, Geometric interpretation of its signature, E.E. Levi's theorem, Connections with Kahlerian geometry, Elementary properties of plurisubharmonic functions.
Introduction to Cohomology: Definition and examples of complex manifolds. The d operators, The Poincare Lemma and the Dolbeaut Lemma, The Cousin problems, Introduction to Sheaf theory.

RECOMMENDED BOOKS

1. J. Morrow and K. Kodaira, Complex Manifolds, Holt, Rinehart and Winston, New York, 1971.
2. L. Hormander, An Introduction to Complex Analysis in Several Variables, D. Van Nostrand, New York, 1966.
3. H. Grauert and K. Fritsche, Several Complex Variables, Springer Verlag, 1976.
4. M. Field, Several Complex Variables and Complex Manifolds, Cambridge University Press, 1982.

MATH-755 THEORY OF GROUP ACTIONS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Survey of theory of group actions, Applications of group actions, Transitivity and k-transitivity, Primitivity, Finite fields and their extensions, Projective line over finite fields, Finite geometries, Projective spaces and their groups, Actions of PGL (n,q) and PSL (n,q) on PG (n-I,q), Simplicity of projective special linear groups over finite fields, Modular group, Parameterization of action of the extended modular group on projective lines over finite fields. Projective and linear groups through actions.

RECOMMENDED BOOKS

1. Coxeter, H.S.M. and Moser, W.O., Generators and Relations for Discrete Groups, Springer-Verlag.
2. J.S. Rose, S., A Course in Group Theory, Cambridge University Press. 1978
3. Johnson, D.L., Presentation of Groups, Cambridge Lecture Notes, 1976.

MATH-756 BANACH LATTICES

Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Vector lattices over the real field, ideals, bands and projections, maximal and minimal ideals vector lattices of finite dimension, duality of vector lattices, normed vector lattices, abstract M-spaces, abstract L-spaces, duality of AL- and AM-spaces.

RECOMMENDED BOOKS

1. Schaeff, H.H., Banch lattices and positive operators, 1971.
2. Schaeff, H.H., Banch lattices and positive operators, 1984.

MATH-757 LINEAR SYSTEM THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Classical Control Theory: Background and review; Highlights of Classical Control Theory; State Variables and the State Space Description of Dynamic Systems.
Linear Vector Space: Fundamentals of Matrix Algebra; Vectors and Linear Vector Spaces; Simultaneous Linear Equations; Eigenvalues and Eigenvectors; Functions of Square Matrices and the Cayley-Hamilton Theorem. Analysis of System: Analysis of Continuous and Discrete Time State Equations; Stability; Controllability and Observability for Linear Systems.
Optimal Control System: The Relationship between state Variable and Transfer Function Descriptions of Systems; Design of Linear Feedback Control Systems; An Introduction to Optimal Control Theory; An Introduction to Nonlinear Control Systems.

Recommended Books

1. W. Brogen, Modern Control Theory, Prentice-Hall (1990)
2. P. Gopal, Modern Control System, New Age Publishers (1994)
3. J. Kailath, Linear Systems, Prentice-Hall (1979)
4. W.J. Rugh, Linear Systems Theory, Prentice-Hall (1995)
5. C. T. Chen, Linear Systems Theory and Design, Oxford University Press (1999)

MATH-758 ELASTRODYNAICS

Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Strain potential, Galerkin vector, vertical load on the horizontal surface of a half space, Love's strain function, Biharmonic functions, Lamb's problem, Cagniard-de Hoop transformation. Transient waves in a layer, forced shear motion of a layer. **Thermoelasticity:** thermal stresses Chadwick's solution of thermoelastic solutions. Piezoelectricity. Tensor formultion of piezoelectricity, elastic waves in a piezoelectric solid, Bleustein-Gnlayev waves.

RECOMMENDED BOOKS

1. Dieulesant D. and Royer, F., Elastic Waves in Solids, John Wiley and Sons, New York, 1980.
2. Fung, Y.C., Foundations of Solid Mechanics, Prentice-Hall, Englewood Cliffs, 1995.
3. Achenbach, Waves Propagation in Elastics Solids, North-Holland, Amsterdam, 1990.

MATH-759 THE CLASSICAL THOERY OF FIELDS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Review of continuum mechanics; solid and fluid media, constitutive equations and conservation equations. The concept of a field. The four dimensional formulation of fields and the stress-energy momentum tensor. The scalar field. Linear scalar fields and the Klein-Gordon equation. Non-linear scalar fields and fluids. The vector field. Linear massless scalar fields and the Maxwell field equations. The electromagnetic energy-momentum tensor. Electromagnetic waves. Diffraction of waves. Advanced and retarded potentials. Multipole expansion of the radiation field. The massive vector (Proca) field. The tensor field. The massless tensor field and Einstein field equations. Gravitational waves. The massive tensor field. Coupled field equations.

RECOMMENDED BOOKS

1. Scipio, L.A., Principles of Continua with Applications, John Wiley, New York, 1969.

2. Landau, L.D., and Lifshitz, M., The Classical Theory of Fields, Pergamon Press, 1980.
3. Jackson, J.D., Classical Electrodynamics, John Wiley, New York, 1975.
4. Misner, C.W., Thorne, K.S., and Wheeler, J.A., Cravitation, W.H. Freeman, 1973.
5. Romen, P., Introduction to quantum field theory, John Wiley, New York, 1969.

MATH-760 PLASMA THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Introduction: Definition of plasma; temperature; Debye shielding, the plasma parameter; criteria for plasmas; introduction to controlled fusion.
Fluid description of plasma: Wave propagation in plasma; derivation of dispersion relations for simple electrostatic and electromagnetic modes.
Equilibrium and stability (with fluid model); Hydromagnetic equilibrium/diffusion of magnetic field into a plasma; classification of instabilities; two-stream instability; the gravitational instability; resistive drift waves.
Space plasma: Atomospheric source of magnetospheric plasma and its temperature; plasma from Jupiter.

RECOMMENDED BOOKS

1. Chen, F.F., Introduction to Plasma Physics, Plenum Press, New York, 1974.
2. Krall, N.A. and Trivelpiece, A.W., Principles of Plasma Physics, McGraw-Hill Book Company, 1973.
3. Glasstone, S., and Lovberg, R.H., Controlled thermonuclear reactions, Van Nartrand Company, 1960.
4. Nishida, A., Magnetospheric Plasma Physics, D. Reidel Publishing Compnay, 1982.
5. Melrose, D.B., Plasma Astrophysics, Gordon and Breach Science Publishers, 1980.

MATH-761 ELECTRODYNAMICS-I
Credit Hours: 03+0

Objectives and Outcomes

Course Contents
Maxwell's equations, Electromagnetic wave equation, Boundary conditions, Waves in conducting and non-conducting media, Reflection and polarization, Energy density and

energy flux, Lorentz formula, Wave guides and cavity Resonators, Spherical and cylinderical waves, Inhomogeneous wave equation, Retarded potentials, Lenard-Wiechart potentials, Field of uniformly moving point charge, Radiation from a gruop of moving charges, Field of oscillating dipole, Field of an accelerated point charge.

RECOMMENDED BOOKS

1. Reitz, J.R., and Milford, F.J., Foundations of Electromagnetic Theory, Addison-Wesley, 1969.
2. Panofsky, K.H., and Philips, M., Classical Electricity and Magnetism, Addison-Wesley, 1962.
3. Corson, D., and Lorrain, P., Introduction to Electromagnetic Fields and Waves, Freeman, 1962.
4. Jackson, D.W., Classical Electrodynamics, John-Wiley

MATH-762 ELECTRODYNAMICS-II
Credit Hours: 03+0

Objectives and Outcomes

Course Contents
General angular and frequency distributions of radiation from accelerated charges, Thomson scattering, Cherenkov radiation, Fields and radiation of localized oscillating sources, Electric dipole fields and radiation, Magnetic dipole and electric quadruple fields, Multipole fields, Multipole expansion of the electromagnetic fields; Angular distributions sources of multipole radiation; Spherical wave expansion of a vector plane wave; Scattering of electromagnetic wave by a conducting sphere.

RECOMMENDED BOOKS

1. Jackson, J.D., Classical Electrodynamics, John-Wiley, 1962.
2. Straton, J.A., Electromagnetic Theory, McGraw-Hill, New York.

MATH-763 MAGNETOHYDRODYNAMICS-I
Credit Hours: 03+0

Objectives and Outcomes

Course Contents
Basic Equations: Equations of electrodynamics, Equations of Fluid Dynamics, Ohm’s law equations of magnetohydrodynamics.

Motion of an Incompressible Fluid: Motion of a viscous electrically conducting fluid with linear current flow, steady state motion along a magnetic field, wave motion of an ideal fluid.
Small Amplitude MHD Waves: Magneto-sonic waves. Alfve's waves, damping and excitation of MHD waves, characteristics lines and surfaces.
Simples Waves and Shock Waves in Magnetohydrodynamics: Kinds of simple waves, distortion of the profile of a simple wave, discontinuities, simple and shock waves in relativistic magnetohydrodynamics, stability and structure of shock waves, discontinuities in various quantities, piston problem, oblique shock waves.

RECOMMENDED BOOKS

1. Cowling, T.G., Magnetohydrodynamics, Interscience Publishers, 1963.
2. Kulikowsky, A.G., and Lyabimov, A.G., Magnetohydrodynamics, A.Wesley, 1965.
3. Alfve's H., and Falthammar, C., Cosmical Electrodynamics, Clarendon Press, 1965.
4. Akhiezer et.al. Plasma Electrodynamics, Pergamon Press, 1975.
5. Kendale and Plumpton, C. Magnetohydrodynamics.
6. Anderson, J.E. Magnetohydrodynamics, Shock Waves, M.I.T. Press, 1975.

MATH-764   MAGNETOHYDRODYNAMICS-II
Credit Hours: 03+0

Objectives and Outcomes

Course Contents
Flow of Conducting Fluid Past Magnetized Bodies: Flow of an ideal fluid past magnetized bodies, Fluid of finite electrical conductivity flow past a magnetized body.
Dynamo Theories: Elsasser's Theory, Bullard's Theory, Earth's field Turbulent motion and dissipation, vorticity anology.
Ionized Gases: Effects of molecular structure, Currents in a fully ionized gas, partially ionized gases, interstellar fields, dissipation in hot and cool clouds.

RECOMMENDED BOOKS

1. Gowling, T.G., Magnetohydrodynamics, Interscience Publishers, 1963.
2. Kulikowskiy, A.G., and Lyubimov, G.A., Magnetohydrodynamics, A.Wesley, 1965.
3. Alfven, H., and Falthammar, C., Cosmical Electrodynamics, Clarendon Press, 1965.

4. Akliezer, A.I., Plasma Electrodynamics, Pergamon Press, 1975.
5. Kendall, P.C., and Plumption, C., Magnetohydrodynamics.
6. Aderson, J.E. Magnetohydrodynamics Shock Waves, M.I.T. Press, Cambridge, 1963.

MATH-765 INTRODUCTION TO ALGEBRAIC CRYPTOGRAPHY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Course overview; History of cryptography, Basic terminology of cryptography, Mathematical background of encryption and decryption, Transposition ciphers, Substitution ciphers. Groups and semigroups, Ring, Subrings, ideals and factor rings, Polynomial rings, Irreducible polynomials, Primitive element, Ringhomorphisms, Euclidean domains and PIDs, Extended Euclidean algorithm, Ring of classes of risidue modulo.
Finite fields, Existence of finite fields of prime order, Polynomials over finite fields, Construction of Galois fields, Examples of Galois fields, the characteristic of a finite field, Algebra of finite fields. Block Ciphers, DES (Data Encryption Standard), AES (Advanced Encryption Standard), Evaluation criteria for AES, Correlations and Walsh Transforms, Cryptographic Criteria for S-Boxes: Nonlinearity, Strict Avalanche Criterion, Balanced Criterionand Bijective Criterion.

RECOMMENDED BOOKS

1. Alexander Stanoyevitch, Introduction to Cryptography with Mathematical Foundations and Computer, Chapman and Hall/CRC, 2010.
2. S.R. Nagpaul and S.K. Jain, Topics in applied Abstract Algebra, Thomson, UK, US, 2005.
3. Douglas Robert Stinson, Cryptography Theory & Practice, CRC Press, 1995.
4. Bruce Schnier, Applied Cryptography, Jon Wiley & Sons, 1996.
5. H.C.A van Tilborg, An Introduction to Cryptology, Kluwer Academic Publisher, Boston, 1988.

MATH-766 LA-SEMIGROUPS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

LA-semigroups and basic results, Connection with other algebraic structures, Medial and exponential properties, LA-semigroups defined by commutative inverse semigroups, Homomorphism theorems for LA-semigroups, Abelian groups defined by LA-semigroups, Embedding theorem for LA-semigroups, Structural properties of LA-semigroups, LA-semigroups as a semilattice of LA-subsemigroups, Locally associative LA-semigroups, Relations on locally associative LA-semigroups, Maximal separative homomorphic images of locally associative LA-semigroups, Decomposition of locally associative LA-semigroups.

RECOMMENDED BOOKS

1. Clifford, A.H. and G.B. Preston., The Algebraic Theory of Semigroups, Vols. I & II, Amer. Math. Soc. Surveys, 7, Providence, R.I.

MATH-767 THEORY OF GROUP GRAPHS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents
Generators and relations, Factor groups, Direct Products, Automorphisms, Finite Presentations of Groups, Tiezte transformations, Coset enumerations, Graphs, Cayley diagram, Schrier's cost diagrams, Coset diagrams for the modular group, Action of the modular group on finite sets, Symmetry in the diagrams, Composition of soset diagrams, Action of the modular group on real projective line, Action of the modular group on finite projective lines over finite fields.

RECOMMENDED BOOKS

1. Coxeter, H.S.M. and Moser, W.O., Generators and relations for discrete Groups, Springer-Verlag.1965.
2. Rose, S., A course in group theory, Cambridge University Press. 1980.
3. Magnus, W., Karrass, A and Solitar, D., Combinatorial group theory, Dover Publications, 1976.

MATH-768 LIE ALGEBRAS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Definitions and Examples of Lie algebras; ideals and quotients; Simple, solvable and nilpotent Lie algebras; radical of a Lie algebra, Semisimple Lie algebras; Engel's nilpotency criterion; Lie's and Cartan theorems; Jordan-Chevalley decomposition; Killing forms; Criterion for semisimplicity; product of Lie algebras; Classification of Lie algebras upto dimension 4; Applications of Lie algebras.

RECOMMENDED BOOKS

1. Humphreys, J.E., Introduction to Lie Algebras and Representation Theory, SpringerVerlag, 1972.
2. Lepowsky, J. and Mccollum, G.W., Elementary Lie algebra Theory, Yale University, 1974.
3. Jacobson, N., Lie algebras, Interscience, New York, 1962.
4. Miller, W. Jr., Symmetry Groups and their applications, Academic Press, 1972.
5. Kramer, D. Stephani, H., Herlt, E and MacCallum, M., Exact Solutions of Einstein's Fields Equations, Cambridge University Press, 1980.
6. O'Neill, B., Semi-Riemannian Geometry, Academic Press, 1983.

MATH-769 ADVANCED MATHEMATICAL PHYSICS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Nonlinear ordinary differential equations, Bernoulli's equation, Riccati equation, Lane-Emden equation, Nonlinear Pendulum, Duffing's equation, Pinney's equation, Perturbation theory, Bogoliubov-Krilov method. Linear partial differential equations, classification, initial and boundary values problems, Fourier analysis, Heat equation, Wave equation, Laplace equation etc. Integral equations, classification, integral transform separable kernels, singular integral equations, Wiencr-Hopf equations, Fredholm theory, series solutions. Variational methods, The Euler-Lagrange equations, Solutions to some famous problems, Sturm-Liouville Problem and variational principles, Rayleigh-Ritz Methods for partial differential equations. Matrix algebra, method of Faddeev, Caley-Hamilton' theorem function of matrices. Functions of matrices, Kronecker and Tensor product, special matrices.

RECOMMENDED BOOKS

1. Stephenson, G and Radmore, P.M.: Advanced Mathematical Methods for Engineering and Science Students (Cambridge University Press, 1990).
2. Riley, K.F., Hobson, P.M. and Bence, S.J.: Mathematical Methods for Physics and Engineering (Cambridge University Press, 2006).
3. Tang, K.T.: Mathematical Methods for Engineers and Scientists Volumes 1,2,3 (Springer, 2007).
4. Stone, M. and Goldbart, P.: Mathematics for Physics (Cambridge University Press, 2009).
5. Arfken, G.B. and Weber, H.J.: Mathematical Methods for Physicists (Academic Press, 2005).

MATH-770 QUANTUM FIELD THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Classical field theory, lagrangian mechanics, variational principle, vibrating stings, classical field theory, Lorentz transformations, Lorentz group, representations of Lorentz group, classical scalar fields, Klein-Gordon equation, complex scalar fields, energy-momentum tensor, electromagnetic field, Maxwell's equations, spinor field, Dirac equation, symmetries and conservation laws, Noether's theorem, translation invariance. Quantization of fields, canonical quantization of fields, quantization of scalar fields, particle interpretation of quantum field theory, normal ordering, non-Hermitian fields. Interacting Quantum Fields, interacting fields, perturbation theory, time ordering, S-matrix, cross section, decay rate of an unstable particle, higher order perturbation theory, Wick's theorem second order perturbation theory, Feynman rules and diagrams, renormalization, mass renormalization, coupling constant renormalization, field renormalization.

RECOMMENDED BOOKS

1. Bogoliubov, N.N. and Shirkov, D.V.: Introduction to the Theory of Quantized Fields (Wiley, 1959).
2. Itzykson, C. and Zuber, J-B.: Quantum Field Theory (McGraw-Hill, 1980).
3. Ryder, L.H.: Quantum Field Theory (Cambridge University Press, 1999).
4. Kaku, M.: Quantum Field Theory (Oxford University Press, 1993).
5. Mandl, F. and Shaw, G.: Quantum Field Theory (Wiley, 1984).
6. Bjorken, J.D. and Drell, S.D.: Relaivistic Quantum Field (McGraw-Hill, 1965).

7. Weinberg, S.: The Quantum Theory of Fields Vol. I, II (Cambridge University Press, 1995).

MATH-771 DESIGN THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Basic definitions and properties, related structure. The incidence matrix, graphs, residual structures. The Bruck-Ryser-Chowla theorem. Singer groups and difference sets. Arithmetical relations and Hadamard 2- designs. Projective and affine planes. Latin squares, nets. Hadamard matrices and Hadamard 20 design. Biplanes, strongly regular graphs. Cameron's theorem and Hadamard 3-desings. Steiner triple systems. The Mathieu groups.

RECOMMENDED BOOKS

1. Hughes, D.R. and Piper, F.C.: Design Theory (Cambridge University Press, 1985).
2. Beth, T., Jungnickel, D. and Lenz, H.: Design Theory (2nd edition)(Cambridge University Press, 2000).
3. Quinn, K., Webb, B., Rowley, C. and Holroyal, F.C.: Combinatorial Designs and their Applications (Chapman and Hall, 1999).
4. Cameron, P.J. and Lint, J. H.: Designs, Graphs, Codes and their links (Cambridge University Press, 1991).
5. Cameron, P.J.: Permutation Groups (Cambridge University Press, 1999).

MATH-772 COMPUTER GRAPHICS
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Introduction to computer graphics and its applications. Overview of raster graphics and transformation pipeline, i.e. transformations between different coordinate systems which involve modelling coordinate system. Device coordinate system. World coordinate system. Normalized coordinate system. Display window coordinate system and screen coordinate system. Graphics output primitives in drawing of lines, polygons, triangles, etc. Draw polylines with different line joining methods. Attributes of graphics primitives like colour, line style and fill style. 2D and 3D transformations and viewing. Describing and using viewing parameters to change the shape of the object, using viewport to change the ratio of clipping window. Differences in viewing and modelling transformations. Window clipping by Cohen-Sutherland algorithm.

RECOMMENDED BOOKS

1. Donald, H. and Baker, M. P.: Computer Graphics with OpenGL (Prentice Hall, 2003).
2. James, D. Foley et al.: Introduction to Computer Graphics (Addison-Wesley, 1993).
3. Richard, S. Wright, Benjamin Lipchak: OpenGL SuperBible (Sams, 2004).

MATH-773 RINGS AND MODULES
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Elementary concepts of rings. Nilpotent element. Units. Prime ideals and maximal ideals. Nilradical and Jacobson radical. Operations on ideals. Extension and contraction. Modules and module homomorphism. Submodules and quotient modules. Operations on submodules. Direct sum and product. Finitely generated modules. Exact sequences. Projective and injective modules. Tensor product of modules. Exactness properties of the tensor product.

RECOMMENDED BOOKS

1. Atiya, M.F. and Macdonal, I.G.: Introduction to Commutative Algebra (Addision Wesley Publishing Company London, 1969).
2. Burton, D.E.: A First Course in Rings and Ideals (Addision Wesley Pub. Co., 1968).
3. Zauki, O.S.P.: Commutative Algebra (London, 1963).
4. Kaplansky, I.: Commutative Rings (Univ. of Chicago Press, 1974).
5. Herstien, I.N.: Non Commutative Rings (Math Association of America).
6. Anderson, F.W. and Fuller, K.R.: Rings and Category of Modules (2nd edition) (Springer-Verlag, New York, 1992).

MATH-774 OPERATOR THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Spectral Theory of Linear operators in Normal spaces: Spectral theory in finite dimensional normed spaces. Basic concepts. Spectral properties of bounded liner operators. Further properties of resolvent and spectrum. Use of complex analysis in spectral theory. Basic algebras. Further properties of Banach algebra.

Compact linear Operators on Normed Spaces and their Spectrum: Compact linear operators on normed spaces. Further properties of compact linear operators. Special properties of compact liner operators on normed spaces.

Spectral Theory of Bounded Self-Adjoint Operators linear operators: Spectral properties. Operations of bounded self adjoint linear operators. Positive operators. Square roots of positive operators. Projection operators.

RECOMMENDED BOOKS

1. Kreyszig, E.: Introductory Functional Analysis with Applications (John Wiley, 1989).
2. Nachbin, L. Introduction to functional Analysis: Branch Spaces and Differential Calculus (Marcel Dekker, Inc. 1981).
3. Rudin, W.: Functional Analysis (McGraw-Hill Inc., 1973).
4. Davis, E.B.: Spectral Theory and Differential Operators (Cambridge University Press, 1995).
5. DeVito, C.L.: Functional Analysis and Linear Operator Theory (Addison Wesley Publishing Co., 1990).

MATH-775 LATTICE THEORY
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Elementary Concepts: Definition of lattice. Some algebraic concepts. Polynomials. Identities. Inequalities. Free lattices. Special elements.

Distributive lattices: Distributive lattices, Characterization and representation theorems. Polynomials and freeness. Congruence relations. Boolean algebra.

RECOMMENDED BOOKS

1. Grazer, G.: Lattice Theory (W.H. Freeman and Company, New York, 1971).
2. Maclane, S. and Birkhoff, G.: Algebra (Macmillan Company, New York, 1967).
3. Davey, B.A. and Priestley, H.A.: Introduction to Lattices and Order (Cambridge University Press, 2002).

4. Blyth, T.S. and Birkhoff, G.: Lattices and Ordered Algebraic Structures (Springer, 2005).
5. Donnellan, T.: Lattice Theory (Elsevier Science Ltd., 1968).

MATH-779 Nature-Inspired Optimization Techniques
Credit Hours: 03+0

Objectives: This course intends to give the required knowledge to MPhil & PhD students for simulation, prediction, identification, and optimization of different complex nonlinear systems using intelligent Nature-inspired optimization algorithms.

Course Outline: Introduction to optimization, Simulated Annealing (SA), Genetic Algorithms (GA), Binary genetic algorithms, Continuous genetic algorithms, Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Firefly Algorithm (FA), Cuckoo Search (CS), Differential Evolution (DE), Artificial Bee Colony Optimization (ABCO), Evolution Strategies (ES) and other state-of-the-art optimization algorithms.

Recommended Books:

- R. L. Haupt, S. E. Haupt, “Practical Genetic Algorithms”, John Wily & Sons, 1998.
- D. Corne, M. Dorigo, “New Ideas in Optimization”, McGraw-Hill, 1999. Ch. 2.
- M. Gen, R. Cheng, "Genetic Algorithms and Engineering Optimization", John Wiley
- and Sons, Inc., 2000.
- E.-G. Talbi, Metaheuristics: From design to implementation. Hoboken, New Jersey, USA: John
- Wiley & Sons, 2009.
- G. Zapfel, R. Braune, and M. Bogl, Metaheuristic search concepts: A tutorial with applications to production and logistics. Heidelberg: Springer Science & Business Media, 2010.
- M. Gendreau and J.-Y. Potvin, Handbook of Metaheuristics. New York, USA: Springer, 2010.
- S. Luke, Essentials of Metaheuristics. Lulu, 2013. Available for free at
- http://cs.gmu.edu/_sean/book/metaheuristics/.
- C. C. Ribeiro and P. Hansen, Essays and surveys in metaheuristics. New York, USA: Springer Science & Business Media, 2012.
- F. Glover and G. A. Kochenberger, Handbook of metaheuristics. Dordrecht: Kluwer Academic Publishers, 2003.
- I. H. Osman and J. P. Kelly, Meta-heuristics: Theory and applications. Norwell, Massachusetts, USA: Kluwer, Academic Publishers, 2012.
- X.-S. Yang, Nature-inspired optimization algorithms. London, UK: Elsevier, 2014.

MATH-780 Engineering Optimization
Credit Hours: 03+0

Objectives: This course will cover practical aspects of optimization methods for solving engineering problems. The methods will include linear and nonlinear programming, integer programming, dynamic programming, network models and an introduction to metaheuristic algorithms. This course will expose students to operations research modelling and basic tools for optimization. It is designed for those students who do not have an engineering background but would like to learn about modelling and optimization concepts. This course will be particularly useful for those who are likely to use these methods in their research and projects.

Learning Outcome: At the end of the course, the student is expected to have the ability to construct mathematical models of real life problems and to use appropriate methods/software to solve the constructed models.

Course Outline: Optimization Algorithms, Metaheuristics, Algorithm Complexity, No Free Lunch Theorems, Double Bridge Problem, Spring Design, Pressure Vessel, Shape Optimization, Optimization of Eigenvalues and Frequencies, Welded Beam, Control Problem, Pontryagin's Principle, inverse finite element analysis and other applications.

Recommended Books:

J. Arora, Introduction to Optimum Design, McGraw-Hill, 1989.
A. Belegundu, A Study of Mathematical Programming Methods for Structural Optimization, PhD thesis, University of Iowa, Iowa, 1982.
X. S. Yang, "Biology-derived algorithms in engineering optimization" (Chapter 32), in: Handbook of Bioinspired Algorithms and Applications (Eds. S. Olariu and A. Y. Zomaya), Chapman Hall/CRC Press, (2005).
Xin-She Yang, Engineering Optimization: An Introduction with Metaheuristic Applications.

MATH-783 Non-Newtonian Fluid Mechanics

Credit Hours: 03+0

Classification of Non-Newtonian Fluids, Rheological formulae (Time-independent fluids, Thixotropic fluids and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity. The Rivlin-Ericksen fluid, Basic equations of motion in rheological models. The linear viscoelastic liquid, Couette flow, Poiseuille flows. The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for

viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.

RECOMMENDED BOOKS:

- John Harris, Theology and Non-Newtonian Flow, Longman Inc, New York 1977.
- W.R. Schowalter, Mechanics of Non-Newtonian fluids, New York, Pergamon Press 1978.
- R.B. Birk, R.c. Armstrong and O. Hassager, Dynamics of Polymeric liquids, Vol. 1, 2nd ed., John Wiley & Sons, New York, NY 1987.
- G Astarita and G. Merrucci. Principles of Non-Newtonian Fluid mechanics, McGraw-Hill 1974.

MATH-790 Fixed Point Theory and its Applications

Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Introduction to Fixed Point theory and its applications. Banach fixed point theorem. Contractive mapping. Kannan and Chatterjea, Zamfirescu fixed point theorems. Ciric's fixed point theorem. Weissinger, Rakotch, Boyd-Wong, Meir-Keeler, Hardy and Rogers and other generalization of contraction mapping. Common fixed point. Fixed point theorem for nonexpansive mappings and related classes of mappings. Quasi nonexpansive mappings. Fixed points of multivalued mappings.

Applications of Banach fixed point theorem. Solution of ODE's. Solution of Variational Inequalities. Solution of Integral Equations. Solution of other functional equations.

RECOMMENDED BOOKS

1. K. Goebel and W.A. Kirk, Topics in Metric Fixed Point Theory
2. , Cambridge University Press, 1990.
3. J. Dugundji and A.Granas, Fixed Point Theory, Polish Scientific Publishers, Warszawa, 1982.
4. V.I. Istratescu, Fixed Point Tjepru. D. Reidel Publication Company, 1981.
5. K. Goebel and S. Reich, Uniform Convexity, Hyperbolic Geometry and Nonexpansive Mapping, Marcel Dekker Inc. 1984.

MATH-793 Iterative Approximation Procedures
Credit Hours: 03+0

Objectives and Outcomes

Course Contents

Course outlines: The Background of Metrical Fixed Point Theory, Fixed Point Iteration Procedures, The Picard Iteration, Fixed Point Formulation of Typical Functional Equations, Banach's Fixed Point Theorem, Theorem of Nemytzki-Edelstein, Quasi-Non expansive Operators, Maia's Fixed Point Theorem, The Krasnoselskij Iteration, The Mann Iteration, The Ishikawa, The Equivalence Between Mann and Ishikawa Iterations, Mann and Ishikawa Iterations with Errors, Modified Mann and Ishikawa Iterations, Ergodic and Other Fixed Point Iteration Procedures, Perturbed Mann Iteration, Viscosity Approximation Methods, Stability of Fixed Point Iteration Procedures, Almost Stability of Fixed Point Iteration Procedures, Weak Stability of Fixed Point Iteration Procedures, Data Dependence of Fixed Points, Sequences of Applications and Fixed Points, Solution of Nonlinear Equations, Nonlinear Equations in Arbitrary Banach Spaces, Nonlinear Equations in Smooth Banach, Nonlinear-Accretive Operator Equations in Reflexive Banach Spaces.

RECOMMENDED BOOKS

1. V. Berinde: Iterative Approximation of Fixed Points, Springer, 2007.
2. R.P. Agarwal, D. O'Regan, D.R. Sahu: Fixed Point Theory for Lipschitzian-type Mappings with Applications, Spring, 2009.