M.Sc Mathematics Previous
Annual Examination 2020

Paper: Real Analysis

Total Marks 100
Pass Marks 40

Time Allowed 04:00 hrs

Note: - Attempt any FIVE questions selecting at least two questions from each section.

Section A

Q 1: (a). Prove that $\sqrt{3}$ is irrational number.

(b). If "x" and "y" are any real numbers with $x < y$, then there exits a rational number "r" such that $x < r < y$.

Q 2: State and prove "Bolzano Weierstrass Theorem".

Q 3: (a). Apply Ratio test to determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}.$$

(b). Apply Cauchy Root test to determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \left(\frac{n}{1+n^3}\right)^n.$$

Q 4: (a). Let $f: [0,1] \to R$ defined by

$$f(x) = \begin{cases} 0, & if\ x\ is\ rational\ number \\ 1, & if\ x\ is\ irrational\ number \end{cases}.$$

Then show that $\lim_{x\to p} f(x)$ does not exists for any $p \in [0,1]$.

(b). Prove that, A function which uniformly continuous on an interval "I" is continuous on "I".

Q 5: (a). State and prove Lagrange's Mean value theorem.

(b). Prove that, A function is derivable at a point is continuous at that point.

Section B

Q 6: (a). Prove that every continuous function is integrable.

(b). If "c" be a point in (a, b) and if f(x) is integrable on [a, b], then prove that

$$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx.$$

Q 7: (a). Prove that the value of each Reimann integrable function f on $[a, b]$ is uniquely determined.

(b). Prove that every Reimann integrable function f on $[a, b]$ is bounded f on $[a, b]$.

Q 8: (a). Prove that if $f(0) = 0$, $f'(x) = \frac{1}{1+x^2}$ then $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$

(b). State and prove the existence theorem for implicit functions.

Q 9: (a). If $f(x)$ is bounded and integrable in (a,b) and $F(x) = \int_a^x f(x)dx$ then $F(x)$ is continuous function on x in (a,b).

(b). If $f(x)$ is continuous on [a, b], then there exists a number c$\in (a, b)$ such that

$$\int_a^b f(x)dx = f(c)(b-a).$$

Math-P–103/20 – *MA*
A/MA/20/ULM

M.Sc Annual Examination 2020 (Final)
Mathematics
Numerical Analysis

Time Allowed: 4 Hours **Total Marks: 100**

Note: Attempt any five questions. All questions carry equal marks. Possession of mobile phone is strictly prohibited.

1. **a.** Find L U decomposition for the matrix $A=\begin{bmatrix} 4 & -2 & 8 \\ -2 & 10 & -10 \\ 8 & -10 & 45 \end{bmatrix}$. (10)

   b. Derive Newton Raphson's method and illustrate its merits and demerits. (10)

2. **a.** Prove that divided differences of order n of x^n are constant, where n is positive integer.(10)

   b. Find the Eigen value and Eigen vectors of $\begin{bmatrix} 1 & -1 & 3 \\ 2 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ (10)

3. **a.** Fit a polynomials corresponding to the following data using Lagrange interpolations formula (10)

x	3	5	7	9	11
$f(x)$	6	24	58	108	174

   b. Solve the system of equations by Jacobi's method correct to four decimal places, (10) using initial guess $x^{(0)} = y^{(0)} = z^{(0)} = 1$, $3x - y + z = 1$, $3x + 6y + 2z = 0$ and $3x+3y+7z=4$.

4. **a.** Apply Newton Forward difference formula to find missing value in the following (10)

x	0	1	2	3	4
f(x)	3	8	15	?	47.

   b. Use the Lagrange interpolations formula to derive the Simpson's $\frac{3}{8}$ rule'. (10)

5. **a.** Solve the initial value problem $y_{k+2} - 3y_{k+1} + 2y_k = 6k - 17$, $y_1 = 13$, $y_2 = 30$. (10)

   b. Find the solution of the initial value problem by using the Euler method. (10)

$$y' = \frac{y-t}{2}, \qquad y(0) = 1 \qquad h = 0.1, 0.5$$

6. **a.** Derive the Gregory Newton formula as a special case of Newton –divided difference formula. (10)

   b. Compare the result of Trapezoidal and Simpson's $\frac{3}{8}$ rule for $\int_0^1 \frac{dx}{1+x^4}$,state which rule is better and why? [redacted] (10)

7. **a.** Compute the first four derivative from the following data in the give table (10)

x	2	5	9	13	15	21
$f(x)$	108243219	121550625	141158164	163047364	174900628	214358884

b. Let the values as given in the following table. Find a polynomial which interpolate the given data

x:	-3	-2	-1	0	1
y:	16	7	4	1	-8

(10)

8. **a.** Apply any iterative method to find approximate root of $x + e^{-x}\cos x = 0$. (10)

b. Find the different norms on matrix A if $A = \begin{bmatrix} 2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{bmatrix}$. (10)

9. **a.** Describe the factorial representations for a polynomial.

b. Apply Tripzodal's rule to calculate $\int_2^3 \frac{\sin x}{x} dx, n = 6$ (10)

M.Sc Annual Examination 2020 (Final)
Mathematics
Advance Analysis

Total Marks: 100

Time Allowed: 4 Hours

Note: Attempt any five questions. All questions carry equal marks. Possession of mobile phone is strictly prohibited.

Q.1. (a) If D_1 and D_2 are disjoints measurable subsets of D with $D = D_1 \cup D_2$, then prove that $\int_D \psi d\mu = \int_{D_1} \psi d\mu + \int_{D_2} \psi d\mu$.

(b) Let X and Y are countable linear ordered sets, then show that the order sum, $X+Y$ is countable.

Q.2. (a). Prove that $[0, 1]$ is equivalent to $[0, 2]$.

(b).Prove that every infinite set has a denumerable subset.

Q.3. (a) Show that union of two measurable set is measurable.

(b) Prove that $E_1 - E_2$ is measurable if E_1, E_2 are measurable.

Q.4.(a). If $E \subset R$, and $\dot{m}(E) = 0$, then E is measurable.

(b). A set consisting on single element is measurable. Find its measure.

Q5. (a) Prove that the function $\exp\left(\frac{\pi}{2}\left(t - \frac{1}{t}\right)\right)$ generates the Bessel functions.

(b) Given a measure space (X, A, μ). If $\{E_n\}_1^\infty$ is an increasing sequence in A, then prove that

$$\lim_{n\to\infty} \mu\{E_n\} = \mu\left\{\lim_{n\to\infty} E_n\right\}.$$

Q.6. (a). Find the measure of $\{a : 3 \le a \le 5\} \cup \{a : -4 \le a \le -2\}$.

(b). Prove that every monotonic function is Lebesuge integrable.

Q.7. (a) Prove that for any mode of partition, if s is lower sum and S is an upper sum, then $s \le S$, where $f(x)$ is bounded function.

(b). If f, g are measurable function defined on E, then prove that $f.g$ is measurable on E.

Q.8. (a). Prove that a bounded and measurable function $f(x)$ *on* $[a, b]$ is Lebesgue integrable.

(b). State and prove Lebesgue theorem on bounded convergence

Q.9. (a). Use Lebesgue dominated convergence theorem to evaluate $\lim_{n\to\infty} \int_1^3 f_n(x)dx$, where

$$f_n(x) = \frac{n^{\frac{3}{2}} x}{1 + (nx)^2}, \; n = 1,2,3..., \; 1 \le x \le 3.$$

(b). If $f(x)$, $g(x)$ are of bounded variation on [a, b], then $f(x) + g(x)$ is also of bounded variation on [a, b] and hence prove that $\| f + g \| \le \| f \| + \| g \|$.

M.Sc Mathematics Final
Annual Examination 2020

Paper: **Differential Equations**

Total Marks 100
Pass Marks 40

Time Allowed 04:00 hrs

Note: - Attempt any FIVE questions selecting at least two questions from each section.

Section I

Q 1: Solve the given differential equations by separation of variables:

(a). $\frac{dy}{dx} = (x+1)^2$ **(b).** $\frac{dy}{dx} = e^{3x+2y}$

Q 2: Determine whether the following differential equation is exact? If Yes, then solve it:

(a). $2xy\,dx + (x^2-1)\,dy = 0$ **(b).** $\frac{dy}{dx} = xy$

Q 3: (a). Let $y_1 = e^x$ be a solution of $y'' - y = 0$ on the interval $(-\infty, \infty)$. Use the reduction of order to find a second solution y_2.

(b). Solve $4y'' + 4y' + 17y = 0$, given that $y(0) = -1, y'(0) = 2$.

Q 4: Solve the following differential equations by "Undetermined coefficient method",

(a). $y'' + 3y' + 2y = 6$ **(b).** $y'' - y' + y = 2\sin 3x$

Q 5: Solve the following differential equations by "Variation of parameters method",

(a). $y'' - 4y' + 4y = (x+1)e^{2x}$ **(b).** $y'' - y' = \frac{1}{x}$

Section II

Q 6: (a). Form PDE by eliminating arbitrary constant a, b in the equation $z = ax + by$.

(b). Form PDE by eliminating arbitrary functions from $z = f(x+it) + g(x-it)$.

Q 7: (a). Find the canonical form of $z_{xx} - 5z_{xy} + 5z_x - z_y + 3z = 0$.

(b). Classify the PDE $e^x u_{xx} + e^y u_{yy} = u$ and transform it into the canonical form.

Q 8: Solve the following Cauchy problems:

(a). $\frac{\delta^2 u}{\delta y \delta x} = x^2 y$ subject to $u(x,0) = x^2, u(1,y) = \cos y$

(b). $\frac{\delta^2 u}{\delta y \delta x} - \frac{\delta u}{\delta x} = 2$ subject to $u(0,y) = 0, u_x(x,0) = x^2$

Q 9: Find the Laplace equation in Spherical Polar Coordinates.

M.Sc Mathematics Previous
Annual Examination 2020

Paper: Topology and Functional Analysis

Total Marks 100

Pass Marks 40

Time Allowed 04:00 hrs

Note: Attempt any FIVE questions at least TWO questions from each section.

SECTION-A

$Q.1$ (a) If $\{\tau\alpha\}_{\alpha\in I}$ is collection of Topologies on X. Then $\bigcup_{\alpha\in I}\tau\alpha$ is a Topology on X. Also give an example to show that $\bigcup_{\alpha\in I}\tau\alpha$ is not a Topology on X.

(b) Show that if U is open in X and A is closed in X then $U - A$ open in X and $A - U$ is closed in X.

$Q.2$ (a) Let A and B be sub-sets of a Topological space X then

I. $(AUB)^o \supseteq A^oUB^o$

II. $(A\cap B)^o = A^o \cap B^o$

(b) Let A be a subset of the Topological space X, Let A' be the set of all limits points of A. Then $\bar{A} = A \cup A'$.

$Q.3$ (a) If X, Y and Z are Topological spaces and $f: X \to Y$ and $g: Y \to Z$ are continuous. Then $gof: X \to Y$ is continuous.

(b) Define $T_1 - space$. Prove that every compact Hausdraff space is normal.

$Q.4$ (a) Define first and second countable space. Prove that any uncountable set X with co finite Topology is neither first countable nor second countable.

(b) Prove that a closed sub-space of a normal space is normal.

$Q.5$ (a) State and Prove Holder's inequality.

(b) Define a Normed Linear Space. Prove that a normed linear space X is a metric space.

SECTION-B

$Q.6$ (a) Prove that a subspace Y of a Banach space X is complete iff the set Y is closed in X.

(b) Prove that in a Banach space, an absolutely convergent series is convergent.

$Q.7$ (a) If a normed space X is finite dimensional, then every linear operator on X is bounded.

(b) Prove that if Y is a Banach space, then $B(X, Y)$ is a Banach space.

$Q.8$ (a) Define a Metric space. Prove that an open sphere in a metric space X is an open set.

(b) Given a set X define (i) $d(x,y) = 1$ if $x \neq y$ (ii) $d(x,y) = 0$ if $x = y$ Then d is a metric space.

$Q.9$ (a) State and Prove Hahn Banach Theorem for normed linear spaces.

(b) State and prove open mapping theorem.

M.Sc Annual Examination 2020 (Final)
Mathematics
Fluid Mechanics

Total Marks: 100

Time Allowed: 4 Hours

Note: Attempt any five questions. All questions carry equal marks. Possession of mobile phone is strictly prohibited.

Q.1.a. Explain the terms Fluid. Discuss it main types. Define gradient, curl and divergence in detail.

b. List important applications of fluid mechanics in engineering, science and technology.

Q.2. (a) Prove that for a compressible fluid the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho.v) = 0$$

(b) Find the equation of the stream line for the two dimensional steady state flow

$u = x^2 y, \quad v = \frac{1}{2} - xy^2$

Q. 3. (a) Discuss the Laminar flow between parallel plates for the Plane Couette flow.

(b) A stream function is given by the expression $\psi = 2x^2 - y$. Find the components u, v and also the resultant velocity at the point $(3,1)$.

Q. 4 a. State and prove Bernoulli's theorem.

b. For $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and a constant vector $\vec{a}$, prove that $div(\vec{a} \times \vec{r}) = 0$

Q.5. Show that the Euler and the Stokes equations are obtained from the Navier- Stokes equations in the limit of small and large Reynolds numbers. Write down flow situations where these limiting behaviors may apply.

Q.6. Calculate the velocity and the acceleration for the one-dimensional, linear motion of the position vector described by

r(t)=ix(t)=ix$_0$(t)

with respect to an observer who

(i) Is stationary at x=x0;

(ii) Is moving with the velocity of the motion;

(iii) Is moving with velocity V in the same direction;

(iv) Is moving with velocity V in the opposite direction.

Q. 7 a. State and prove Gauss's divergence theorem.

b. Show that d = d$\vec{r}$.$\vec{\nabla}$.

Q.8. a. Show that the following velocity field is a possible case of irrotational flow of an incompressible fluid. $u = yzt, \; v = zxt, \; w = xyt.$

b. Derive equations of motion for fluid flow.

Q.9. a The stream function in two dimensional motion is given by $\psi = cr^2\theta$ where r, θ are the polar coordinates. Find the vorticity and velocity.

b. In two dimensional motions find the polar coordinates of the velocity.

M.Sc Mathematics Final
Annual Examination 2020

Paper: Advanced Group Theory

Total Marks 100
Pass Marks 40

Time Allowed 04:00 hrs

Note. Attempt any five questions. All questions carry equal marks.

Q1. **a)** Let K be kernel of the homomorphism θ from the group G onto the group H.

Prove that H is isomorphic to G/K.

b) Prove that Conjugacy is an equivalence relation on a group G.

Q2. **a)** Every group of order P^2 is abelian, where P is a prime number.

b) Prove that every characteristic subgroup of a group is normal in that group.

Q3. **a)** For any two groups H and K prove that $H \times K$ isomorphic to $K \times H$.

b) Prove that every finite group G has a syllow p-subgroup corresponding to a prime P dividing the order of G.

Q4. **a)** State and prove Jordan Holder Theorem.

b) White the composition series for Q_8 and show that they are isomorphic.

Q5. **a)** Every subgroup and factor group of a solvable group is solvable.

b) Prove that the direct product of finite number of nilpotent group is nilpotent.

M.Sc Mathematics Final
Annual Examination 2020

Paper: Number Theory

Total Marks 100
Pass Marks 40

Time Allowed 04:00 hrs

Note: Attempt any FIVE questions at least TWO questions from each section.

$Q.1$ (a) Prove that there exists infinitely many positive integers n such that $4n^2+1$ is divisible both by 5 and 13.

(b) Prove that for positive integer m and $a>1$ we have

$$\left(\frac{a^m-1}{a-1}, a-1\right) = (a-1, m)$$

$Q.2$ (a) Prove that there exist infinitely many pairs of positive integers x, y such that

$$x(x+1)|y(y+1), \quad x \nmid y, \quad x+1 \nmid y, \quad x \nmid y+1, \quad x+1 \nmid y+1$$

And find the least such pair.

(b) Prove that for every integer k the numbers $2k+1$ and $9k+4$ are relatively prime, and for numbers $2k-1$ and $9k+4$ find their greatest common divisor as a function of k.

$Q.3$ (a) Prove that if a, b, c are three different positive integers, then there exist infinitely many positive integers n such that $a+n, b+n, c+n$ are pairwise relatively prime.

(b) Find all rectangular triangles with integer sides forming an arithmetic progression.

$Q.4$ (a) Find all arithmetic progressions with difference 10 formed of more than two primes.

(b) Prove that Mobios function is multiplicative

$Q.5$ (a) Prove that norm of an algebraic integer is a rational integer.

(b) Find the units of $R|\sqrt{-5}|$ and $R|\sqrt{-1}|$.

$Q.6$ (a) Prove that there exist arbitrarily long arithmetic progressions formed of different positive integers such that every two terms of these progressions are relatively prime.

(b) Prove that for every positive integer k the set of all positive integers n whose number of positive integer divisors is divisible by k contains an infinite arithmetic progression.

$Q.7$ (a) Prove by elementary means that the equation $3x^2+7y^2+1=0$ has infinitely many solutions in positive integers x, y.

(b) Prove that the equation $x(x+1)=4y(y+1)$ has no solutions in positive integers x, y but has infinitely many solutions in positive rationals x, y.

$Q.8$ (a) Prove that the equations

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 1$$

Has no solutions in positive integers x, y, z.

(*b*) Find all solutions of the equation

$$x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$$

In integers x.

M.Sc Mathematics Previous
Annual Examination 2020

Paper: Algebra (Group Theory and Linear Algebra)

Total Marks 100

Pass Marks 40

Time Allowed 04:00 hrs

Note: Attempt any FIVE questions at least TWO questions from each section.

SECTION-A

$Q.1$ (a) A subgroup H of a cyclic group G is itself Cyclic.

(b) Every group G is isomorphic to a permutation group.

$Q.2$ (a) Let H be a subgroup of G and $x \in G$. Then $x^{-1}Hx = \{x^{-1}hx : \ h \in H\}$ is a subgroup of G.

(b) Prove that H is a normal subgroup of G iff every right cosets Hx is also a left coset xH.

$Q.3$ (a) Suppose H and K are finite subgroup of a group G. Then

$$|HK| = \frac{|H|.|K|}{|H \cap K|}$$

(b) Let φ be a homomorphism of G into $\overline{G}$. Then $\overline{H}$ the homomorphic image of G in $\overline{G}$ is a subgroup of $\overline{G}$.

$Q.4$ (a) Let G be a group and N a subgroup of G. N is normal in G iff the product of any two right cosets of N is a right coset of N.

(b) Suppose φ is a homomorphism of a group G onto a group $\overline{G}$ and K is a kernel of φ. Then $\overline{G}$ is isomorphic to G/K.

$Q.5$ (a) The sylow P-subgroups of a finite group G are conjugate.

(b) if G is a nilpotent p-group of class $c > 1$ then $[G : \ H_{c-1}]$ is divisible by p^2.

SECTION-B

$Q.6$ (a) Let V be a vector space and W a subset of V. Then W is a subspace of V if and only if the following three conditions hold for the operations defined in V

I. $0 \in W$
II. $x + y \in W$ whenever $x \in W$ and $y \in W$
III. $cx \in W$ whenever $c \in F$ and $x \in W$

(b) Define direct sum. The span of any subset S of a vector space V is a subspace of V that contains S must also contain the span of S.

$Q.7$ (a) Let S be a linearly independent subset of a vector space V, and let $\boldsymbol{v}$ be a vector in V that is not in S. Then $S \cup \{\boldsymbol{v}\}$ is linearly dependent iff $\boldsymbol{v} \in span(S)$.

(b) State and prove Replacement Theorem.

$Q.8$ (a) Let V and W be vector spaces, and let $T: V \to W$ be linear. Then T is one-to-one if and only if $N(T) = \{0\}$.

(b) Let V and W be vector space over F, and suppose that $\{v_1, v_2, v_3, \cdots, v_n\}$ is a basis for V, for $\{w_1, w_2, w_3, \cdots, w_n\}$ in W, there exist exactly one linear transformation $T: V \to W$ such that

$$T(v_i) = w_i , \qquad i = 1,2,3, \cdots, n$$

$Q.9$ (a) Let V and W be finite-dimensional vector spaces with ordered bases β and γ respectively, and let $T, U: V \to W$ be linear transformations. Then

I. $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$

II. $[\alpha T]_\beta^\gamma = \alpha[T]_\beta^\gamma$ $\quad$ *for all scalar* α.

(b) Let V, W and Z be vector spaces over the same field F and let $T: V \to W$ and $U: W \to Z$ be linear. The $UT: V \to Z$ is linear.

M.Sc Mathematics Final
Annual Examination 2020

Paper: Theory of Optimization

Total Marks 100
Pass Marks 40

Time Allowed 04:00 hrs

Note: Attempt any FIVE questions Each carry equal Marks.

SECTION-A

$Q.1$ (a) Find the solution of the following system of equations:

$$3x_1 + x_2 + x_3 = 8$$
$$3x_1 - x_2 - x_3 = -3$$
$$x_1 + 2x_2 - x_3 = 2$$

(b) Consider the solution of the following two variables minimization problem:

$$f = x_1^2 + x_2^2 - 2x_1 + 3x_2$$
$$x_1 + x_2 + 5 \le 0$$
$$x_1 + 2 \le 0$$

$Q.2$ Obtain all Points satisfying KT conditions for the following optimization problem:

$$Minimize \quad f(x,y,z) = 1/(x^2 + y^2 + z^2)$$
$$Subject\ to \begin{pmatrix} x^2 + 2y^2 + 3z^2 = 1 \\ x + y + z = 0 \end{pmatrix}$$

$Q.3$ (a) Use the method of Lagrange multipliers to find the minimum value of $f(x,y) = x^2 + 4y^2 - 2x + 8$ subject to the constraint $x + 2y = 7$

(b) The golf ball manufacturer, Pro-T, has developed a profit model that depends on the number x of golf balls sold per month (measured in thousands), and the number of hours per month of advertising y, according to the function

$$z = f(x,y) = 48x + 96y - x^2 - 2xy - 9y^2$$

Where z is measured in thousands of dollars. The budgetary constraint function relating the cost of the production of thousands golf balls and advertising units is given by $20x + 4y = 216$. Find the values of x and y that maximize profit and find the maximum profit.

$Q.4$ (a) Prove that given n distinct real values $x_1, x_2, x_3, \cdots, x_n$ and n real values $y_1, y_2, y_3, \cdots, y_n$ (not necessarily distinct), there is a unique polynomial P with real coefficients satisfying $P(x_i) = y_i$ such that $\deg(P) < n$.

(b) Using Lagrange interpolation to find a polynomial P of degree < 4 satisfying

$$P(1) = 1, \ P(2) = 4, \ P(3) = 1, \ P(4) = 5$$

What are the polynomials $P_1(x),\ P_2(x),\ P_3(x),\ P_4(x),\ P(x)$?

$Q.5$ (a) Using Simplex Method to solve the Problem:

$$Minimize \quad f = 5x_1 - 3x_2 - 8x_3$$
$$Subject\ to \begin{pmatrix} 2x_1 + 5x_2 - x_3 \le 1 \\ -2x_1 - 125x_2 + 3x_3 \le 9 \\ -3x_1 - 8x_2 + 2x_3 \le 4 \\ x_i \ge 0, \ i = 1,2,3 \end{pmatrix}$$

(b) Solve the following LP Problem using the tableau form of the Simplex method.

$$Maximize \quad f = -7x_1 - 4x_2 + 15x_3$$

$$\text{Subject to} \begin{pmatrix} \frac{x_1}{3} - \frac{32x_2}{9} + \frac{20x_3}{9} \le 1 \\ \frac{x_1}{6} - \frac{13x_2}{9} + \frac{5x_3}{18} \le 2 \\ \frac{2x_1}{3} - \frac{16x_2}{9} + \frac{x_3}{9} \ge -3 \\ x_i \ge 0,\ i = 1,2,3 \end{pmatrix}$$

$Q.6$ (a) State and prove the Necessary Condition of Euler-Lagrange Theorem.

(b) Find the valid KT Points if Possible

$$f(x,y) = -x$$
$$y - (1-x)^3 \le 0$$
$$-x \le 0$$
$$-y \le 0$$

$Q.7$ Use KT-Condition to find the Solution of the following two variable minimization problems

$$f(x,y) = -x - y$$
$$\begin{pmatrix} x + y^2 - 5 \le 0 \\ x - 2 \le 0 \end{pmatrix}$$

$Q.8$ (a) Find the minimum point, two distinct local maximum points and two inflection points of the function

$$f(x,y) = 25x^2 - 12x^4 - 6xy + 25y^2 - 24x^2y^2 - 12y^4$$

(b) The following function of four variables shows three different local minimum points. If this function was from a practical optimization problem, finding a global minimum would be important.

$$f = x_1^4 + x_1^2(1 - 2x_2) - 2x_2 + 2x_2^2 - 2x_3 + x_3^2 + x_3^4 + 2x_1(-2 + x_4) - 2x_3^2x_4 - 4x_4 + 2x_4^2$$

Q.

M.Sc Mathematics Final
Annual Examination 2020

Paper: Mathematical Statistics

Time Allowed 04:00 hrs

Total Marks 100
Pass Marks 40

Note: Attempt any FIVE questions at least TWO questions from each section.

SECTION-A

Q.1 (a) Two events A and B are such that $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Show that (i) A and B are independent events? (ii) A and B are mutually exclusive events (iii) Find $P(A \cap B)$ and $P(B)$.

(b) Of 12 eggs in a carton, 2 are bad. From these, 4 are chosen at random. What are the probabilities that (i) exactly one egg is bad, (ii) at least one egg is bad.

Q.2 (a) Prove that the binomial distribution is a limiting case of Hyper geometric distribution.

(b) State and prove CHEBYSHEV'S inequality.

Q.3 (a) Find the m.g.f. of the negative Binomial distribution. Show that its mean is smaller than variance.

(b) Prove that Poisson distribution is a limiting case of binomial distribution.

Q.4 (a) Find the mean, variance and m.g.f. of the geometric distribution.

(b) Let X be a random variable with probability distribution

X	-1	0	1	2	3
$f(X)$	0.125	0.5	0.20	0.05	0.125

Find $E(X^2)$ and $Var(X)$.

SECTION-B

Q.5 (a) Compute the least squares regression equation of Y on X for the following data:

X	5	6	8	10	12	13	15	16	17
Y	16	19	23	28	36	41	44	45	50

(b) Calculate the product moment co-efficient of correlation between X and Y from the following data:

X	1	2	3	4	5
Y	2	5	3	8	7

Q.6 (a) Find the co-efficient of rank correlation from the following rankings of 10 students in State and Maths.

M.Sc Mathematics Previous
Annual Examination 2020

Paper: Complex Analysis and Differential Geometry

Total Marks 100

Pass Marks 40

Time Allowed 04:00 hrs

Note: Attempt any FIVE questions at least TWO questions from each section.

SECTION-A

$Q.1$ (a) Define analytic function. Let $f(Z) = |Z|^2$ show that f is continuous at all $Z \in ₵$, but has a derivative only at the origin.

(b) Prove that necessary condition for the function f defined by

$f(z) = u(x,y) + iv(x,y)$ to be analytic in a domain D is that the four partial derivatives exists and satisfy Cauchy-Riemann conditions at each point of D.

$Q.2$ (a) If u and v are harmonic conjugate of each other then u and v are both constant functions.

(b) Show that the given function u is harmonic in $D = ₵$ and determine its harmonic conjugate

$u(x,y) = \frac{2x}{x^2+y^2}, D = ₵ - \{(0,0)\}$.

$Q.3$ (a) Suppose that the power series $\sum_{n=0}^{\infty} a_n(z - z_o)^n$ has a non-zero radius of convergence R. Then for any circle Γ with center at z_o and radius of convergence $\gamma < R$, the power series converges uniformly with in and on Γ.

(b) Explained in a Taylor series, the function f define by

$f(z) = logz = log|z| + iArg(z)$, about the point $z_o = -1 + i, (-\pi < Arg(z) \leq \pi)$

$Q.4$ (a) Define simple closed contour, and evaluate $\int_c z^2 dz$, when

(i) C is the contour OB from $z = 0$ to $z = 2 + i$.

(ii) C is the contour OAB.

(b) State and prove Goursat's lemma.

$Q.5$ Evaluate the following:

(i) $\int_0^{2\pi} \frac{d\theta}{a+bSin\theta}$, $a > |b|$.

(ii) $\int_0^{\infty} \frac{x^2}{(x^2+1)^2} dx$

(iii) $\int_c \frac{z^2+4}{z^2+1} dz$, where C is the circle $|z| = 2$ described in the +ve direction.

SECTION-B

$Q.6$ (a) Define Curvature and Torsion of a curve. Find curvature and torsion o the curve, $x = acos\theta,\ y = asin\theta,\ z = a\theta cot\beta$

(b) If the tangent and binormal at a point of curve make angle θ and ϕ respectively with a fixed direction, show that $\frac{sin\theta}{sin\phi}.\frac{d\theta}{d\phi} = -\frac{K}{T}$.

$Q.7$ (a) Prove that for any curve, $[\vec{t}', \vec{t}'', \vec{t}'''] = K^5 \frac{d}{ds}\left(\frac{T}{K}\right)$

$$[\vec{b}', \vec{b}'', \vec{b}'''] = T^5 \frac{d}{ds}\left(\frac{K}{T}\right)$$

(b) Define Osculating plane and find its equation at any point of a curve.

$Q.8$ (a) Spheres of constant radii b have their centre on a fixed circle

$$x^2 + y^2 = a^2,\ z = 0$$

(b) Calculate the fundamental magnitudes for the surface

$x = (b + asinu)cosv,\quad y = (b + asinu)sinv,\quad z = acosu.$

$Q.9$ (a) Prove that necessary and sufficient condition for the lines of curvature to be parametric curve is that $F = 0,\ M = 0.$

(b) If K_n is the normal curvature of a curve at a point on the surface then

$$K_n = \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2}$$

M.Sc Annual Examination 2020 (Previous)
Mathematics
Paper= Mechanics

Time Allowed: 4 Hours **Total Marks: 100**

Note: Attempt any five questions. All questions carry equal marks. Possession of mobile phone is strictly prohibited.

Q.1.a. Show that the work done on the particle in moving it from a position P_1 to P_2 under a conservative field of force is the difference between the potential energies of the particle at P_1 and respectively. (10)

b. Derive the Normal and tangential components of velocity and acceleration. (10)

Q.2. (a) Show that the eigenvalues of a second order tensor A_{ij} are independent of the coordinate system. (10)

(b) Expresses angular momentum in tensor notation. (10)

Q. 3. a. Prove that $\nabla.(\nabla f \times \nabla g) = 0$ (10)

b. Show that div (cerl$\vec{q}$)= $\vec{\nabla}(\vec{\nabla} \times q) = [\vec{\nabla}.\vec{\nabla} \times \vec{q}]$=0 (10)

Q. 4 A system of forces act on a plate in the form of an equailateral triangle of side $2a$. The moments of the forces about three vertices are G_1, G_2, G_3 respectively, Find the magnitude of the resultant. (10)

b. If forces $p\,\vec{AB}$, $q\,\vec{CB}$, $r\,\vec{CD}$, $s\,\vec{AD}$ acting along the sides of a plane quadrilateral are in equilibrium show that $pr = qs$ (10)

Q.5. a. Derive the Euler equations for rigid body motion in a force field. Use these two obtained a complete solution of problem of free rotation of a symmetrical rigid body. (10)

b. Describe how Euler equations can be used to discuss the motion of a solid cylinder rolling down on an inclined plane. (10)

Q.6 Calculate the velocity and the acceleration for the one-dimensional, linear motion of the position vector described by (20)

$$r(t)=ix(t)=ix_0(t)$$

with respect to an observer who

(i) Is stationary at $x=x0$;

(ii) Is moving with the velocity of the motion;

(iii) Is moving with velocity V in the same direction;

(iv) Is moving with velocity V in the opposite direction.

Q. 7 a. State and prove parallel axis theorem. (10)

b. Find the moment of inertia of solid homogeneous cube with edge length 2a about the concurrent axes and also there product of inertia. (10)

to canonical form and hence solve it.

Q.8.(a). The vector **v** has the representation $\mathbf{v} = (x2 + y2)\,\mathbf{i} + xy\,\mathbf{j} + \mathbf{k}$ in Cartesian Coordinates. Find the representation of **v** in cylindrical coordinates that share the same origin. (10)

(b) Prove the following identity for the vector triple product (10)

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

Q.9. (a) Sketch the vector $\mathbf{u} = 3\,\mathbf{i} + 6\,\mathbf{j}$ with respect to the Cartesian system. Find the dot products of **u** with the two basis vectors **i** and **j** and compare them with its components. Then, show the operation which projects a two-dimensional vector on a basis vector and the one projecting a three-dimensional vector on each of the mutually perpendicular planes of the Cartesian system. (10)

(b) In two dimensional motions find the polar coordinates of the velocity. (10)

M.Sc Mathematics Final
Annual Examination 2020

Paper: **Methods of Mathematical Physics**

Total Marks 100
Pass Marks 40

Time Allowed 04:00 hrs

Note: Attempt any FIVE questions at least TWO questions from each section.

SECTION-A

$Q.1$ (a) Find the solution to $y'' + xy' + y = 0$ with $y(0) = 0$ and $y'(0) = 1$.

(b) Find the first three nonzero terms of two linearly independent solutions to

$xy'' + 2y = 0$.

$Q.2$ (a) Solve

$$(x^2 - 1)y'' + xy' - y = 0$$

(b) Prove that the derivative of $J_v(x)$ with respect to x can be expressed by $J_{v-1}(x)$ or $J_{v+1}(x)$ by the formulas

i. $[x^v J_v(x)]' = x^v J_{v-1}(x)$
ii. $[x^{-v} J_v(x)]' = -x^{-v} J_{v+1}(x)$

$Q.3$ (a) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial x} = (x + y)z$$

(b) If $u_i(x_1, x_2, x_3, \cdots, x_n, z) = c_i \quad (i = 1,2,3, \cdots, n)$ are independent solutions of the equations

$$\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = \frac{dx_3}{P_3} = \cdots = \frac{dx_n}{P_n} = \frac{dz}{R}$$

Then the relation $\phi(u_1, u_2, u_3, \cdots, u_n) = 0$, in which the function ϕ is arbitrary is a general solution of partial differential equation

$$P_1 \frac{\partial z}{\partial x_1} + P_2 \frac{\partial z}{\partial x_2} + \cdots + P_n \frac{\partial z}{\partial x_n} = R$$

$Q.4$ (a) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

(b) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form and hence solve it.

$Q.5$ (a) Solve the Cauchy Problem

$$u_t + u_{xx} = 0, \quad u(x,0) = h(x)$$

(b) Reduce to canonical form and find the general solution

$$y^2 u_{xx} - 2yu_{xy} + u_{yy} = u_x + 6y$$

SECTION-*B*

$Q.6$ (a) Find Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

(b) Find Fourier Transform of

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}, & x = 0 \\ e^{-x}, & x > 0 \end{cases}$$

$Q.7$ (a) If $\vec{F} = P\vec{\imath} + Q\vec{\jmath}$ is a twice continuously differentiable vector field on R, then

$$\int_C \vec{F}.\vec{N} ds = \int\int_R (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}) dxdy$$

(b) State and Prove Green's Theorem.

$Q.8$ (a) Using Green's Theorem to Evaluate

$$\oint_c (y - sinx)dx + cosxdy$$

Where c is the triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2}{\pi}x$

(b) If $\vec{F} = -4y\hat{k}$, then find the surface integral over the rectangle bounded by the lines $x = \pm a$, $y = 0$ & $y = b$

$Q.9$ (a) Discuss maxima and minima for $x^3 + y^3 - 3xy$

(b) Find the extreme points of $f(x,y,z) = 2x + 3y + z$ with conditions

$x^2 + y^2 = 5$ & $x + z = 1$.